Welfare Implications of Debt and Transfers in a Low Safe Rate Environment

Julien H. Acalin

Johns Hopkins University

First draft: July 1, 2019. This draft: October 20, 2019.

Abstract: This paper discusses the welfare implications of inter-generational transfers and debt rollovers in a stochastic overlapping-generations (OLG) economy where the growth rate is higher than the safe rate but lower than the average marginal product of capital. Such an economy is *dynamically inefficient* as a social planner could generate a Pareto welfare improvement by introducing a policy consisting of a combination of inter-generational transfers. *Dynamic inefficiency* does not necessarily imply that the economy has over-accumulated capital: one particular combination of debt rollover and wages subsidy would actually lead to an *increase* in the steady-state level of capital.

Keywords: Stochastic OLG, Intergenerational transfers, Debt, Welfare, Python

Author's Note: The code is provided in a companion Jupyter notebook. I am particularly grateful to Christopher Carroll for guidance, to Olivier Blanchard for insightful conversations, and to Gonzalo Huertas for sharing his Matlab codes. All errors are mine.

1 Introduction

A decade after the onset of the Great Recession, interest rates and GDP growth remain, and are expected to remain, historically low in most advanced economies. The secular stagnation hypothesis, most forcefully articulated by Summers (2013), argues that this era of low interest rates and modest growth may represent a *new normal*. In such environment, monetary and fiscal policy face fresh challenges. First, low interest rates make the effective lower bound more likely to bind and imply a more important role for fiscal policy. Second, to the extent that demand is too low to deliver output at potential, higher deficits may be needed to sustain output. Consequently, discussions on debt and deficits have regained momentum in the public debate. This raises two questions about the feasibility and desirability of higher deficits and debt.

Regarding feasibility, can the government run a budget deficit and rollover its debt forever? The historical behavior of interest rates and growth rates in the U.S suggests there is a high probability that the answer is positive. Figure A shows that, assuming a 25-year rolling window, average short-term and long-term interest rates have been lower than the average growth rate for about 94% and 63% of the time after WWII. Put differently, the probability of a successful debt rollover starting at any point in time after WWII and lasting for a 25-year period was between 63% and 94%[1]. Proponents of the modern monetary theory (MMT) recently got a lot of attention by arguing that the U.S. government should issue more debt and simply rollover its debt to stimulate the economy and increase welfare. However, such deficit gambles, as coined by Ball, Elmendorf and Mankiw (1998), are clearly risky. A sudden rise in interest rates relative to growth with a large stock of debt could quickly become quite costly to service and lead to default.

Assuming it is feasible to sustain higher deficits, is it necessarily desirable to implement such policy? Government debt is often criticized as having crowding-out effect on capital accumulation, because decreasing investment is assumed to be detrimental to economic growth. Yet, this need not be the case. In the classical deterministic overlapping generations (OLG) model originally proposed by Diamond (1965), building on Samuelson (1958), higher public debt can lead to higher steady-state consumption and be welfare improving. This result hold

[1]See Mehrotra (2018) for a more detailed discussion.

as long as the interest rate is lower than the growth rate of the economy. An economy in such situation is called *dynamically inefficient*: higher debt accumulation raises consumption per capita, while reducing capital accumulation. However, the analysis differs when uncertainty is introduced. Abel et al. (1989) extend the overlapping-generations model to account for more general stochastic production functions and risk varying interest rates. They conclude that if capital income always exceeds investment, then the economy is in a *dynamically efficient* state.

In a recent contribution, Blanchard (2019) emphasizes that a higher debt level has two effects on welfare: an effect through reduced capital accumulation as described above, and also an indirect effect, through the induced change in the returns to labor and capital. The welfare effect through lower capital accumulation depends on the safe rate: it is positive if, on average, the safe rate is less than the growth rate. The welfare effect through the induced change in returns to labor and capital depends instead on the average marginal product of capital. It is negative if, on average, the marginal product of capital exceeds the growth rate. Thus, in the current situation where it indeed appears that the safe rate is less than the growth rate, but the average marginal product of capital exceeds the growth rate, the two effects have opposite signs, and the effect of the transfer on welfare is ambiguous. Put simply, the net effect of debt or transfers may be positive if the safe rate is sufficiently low and the average marginal product is not too high.

This paper builds on and extends the analysis developed in Blanchard (2019). First, the paper explores different policy options and shows that the introduction of a wage subsidy can improve long-term welfare in the current low safe rate environment. Second, the paper shows that a carefully designed combination of policies can lead to a Pareto welfare improvement in the current low safe rate environment. Put another way, the economy is *dynamically inefficient* in the current environment, despite the fact that capital income always exceeds investment. Third, the paper shows that Pareto welfare improving policies may not necessarily lead to a decrease in steady-state capital. This challenges the view that *dynamic inefficiency* is generally associated with over-accumulation of capital.

The main conclusions are as follows.

Section 3 shows that the specification of the production function and the design of the transfer scheme are crucial to assess the long-run welfare implications of debt and transfer policies. As shown in Blanchard (2019), if the transfer is deterministic then both average risk-free and risky rates matter if the production function is Cobb-Douglas, while only the risk-free rate matters to a first order approximation if the production function is linear. Yet, this paper shows that if the transfer is stochastic then only the average risky rate matters, independently of the production function specification. This section also emphasizes the tension between policies that would favor early generations at the detriment of future generations, and policies that would produce the opposite outcome. On the one hand, the introduction of a pay-as-you-go (PAYGO) system with fixed transfers (i.e. with a stochastic tax rate) could improve steady-state welfare if the growth rate is sufficiently above the average safe rate and sufficiently close to the average risky rate. However, for most combinations of average rates, such policy would increase the welfare of current generations but decrease the welfare of future generations. On the other hand, if the average risky rate is sufficiently above the growth rate of the economy, the introduction of a PAYGO system with a fixed negative tax rate (i.e. a wage subsidy) would increase the welfare of future generations but decrease the welfare of the current *old* generation which would finance the initial subsidy but not benefit from it.

Section 4 shows that the economy is likely to be *dynamically inefficient* in the current low rate environment. The simulations show that a combination of a policy with fixed transfers and a wage subsidy can generate a Pareto welfare improvement. Moreover, the simulations show that a combination of a debt rollover policy and a wage subsidy can also generate a Pareto welfare improvement. Interestingly, the latter combination of policies leads to an increase in steady state capital. Put another way, such an economy is *dynamically inefficient* in the sense that there exists a combination of policies that leads to a Pareto improvement. Yet, depending on the combination of policies there could be either a long-term crowding-out or crowding-in of capital. Finally, an extended debt rollover —a policy that consists in issuing an additional amount of new debt every period and rolling over the the entire stock of debt— combined with a wage subsidy also leads to a Pareto improvement, independently of the production function specification.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies the long-term welfare implications of inter-generational transfers. Section 4 studies the short-term transitional welfare implications of debt rollovers and transfers, and section 5 concludes.

2 The Stochastic OLG Model

The description of the stochastic overlapping generations (OLG) model follows Blanchard (2019). The model is a stochastic variant of the canonical two-period OLG model, like the one originally proposed by Diamond (1965), building on Samuelson (1958). This model allows to study the effects of different government policies on the capital stock and the welfare of different generations. More specifically, the model is used to examine the transitional and steady-state welfare effects of the introduction of inter-generational transfers and debt rollovers. A detailed derivation of the model is provided in the appendix.

2.1 Households

Environment. Time is discrete. The economy is closed. Two overlapping cohorts of equal size are alive at any point in time. Following Blanchard (2019) households live for two periods of 25 years each, working in the first when *young*, and retiring in the second when *old*. They have separate preferences vis-à-vis intertemporal substitution and risk.

Preferences. Households maximize their expected utility, given by the Epstein-Zin-Weil specification:

$$\mathbb{U}_t = (1-\beta)\ln C_t^y + \beta \frac{1}{1-\gamma} \ln \mathbb{E}[(C_{t+1}^o)^{1-\gamma}] \tag{1}$$

Where C_t^y and C_{t+1}^o respectively denote consumption when *young* and *old*. Given the log utility specification, the elasticity of substitution is equal to 1. The parameter γ is the coefficient of relative risk aversion. This specification allows for different risk aversion coefficients while maintaining the elasticity of substitution unchanged. Note that in this specification the income and substitution effects perfectly offset each other.

Budget constraints. Households work and receive a wage when *young*. They consume part of their first-period income and save the rest to finance their consumption when *old*. Labor supply is assumed to be inelastic and normalized to 1: $L_t = 1\ \forall t$. Households do not work when *old*. The economy is subject to aggregate productivity shocks, but no idiosyncratic or cohort specific shocks.

The budget constraints are:

$$C_t^y + I_t + D_t = W_t + X - T_t - \theta_t \tag{2}$$

$$C_{t+1}^o = R_{t+1} I_t + R_t^f D_t + T_{t+1} \tag{3}$$

Where I_t is investment in physical capital, D_t is investment in safe public debt, W_t is the wage, X is an initial non-stochastic endowment[2], T_t and T_{t+1} denote inter-generational transfers (transfers can be stochastic or not), θ_t is a tax raised on the *young* in case of a debt rollover failure, and R_{t+1} and R_t^f denote respectively the return to physical capital and the risk-free rate.

2.2 Firms

Production. Output is produced by competitive firms using capital and labor. Production is constant elasticity of substitution (CES) in labor and capital, and subject to technological shocks:

$$Y_t = A_t F(K_{t-1}, L_t) \tag{4}$$

where A_t is aggregate total factor productivity. A_t is independently and identically distributed and follows a log-normal distribution: $\log(A) \sim \mathcal{N}(\mu, \sigma)$. Note that there is no technological progress, nor population growth, so the average growth rate of the economy is normalized to zero.

More specifically, the production function is either Cobb-Douglas or linear. In the Cobb-Douglas case:

$$Y_t = A_t K_{t-1}^{\alpha} L^{1-\alpha} \tag{5}$$

In the linear case:

$$Y_t = A_t(\alpha K_{t-1} + (1-\alpha)L) \tag{6}$$

where $L = 1$.

[2]Given that the wage follows a log-normal distribution and thus can be arbitrarily small, the fixed endowment received by the *young* is needed to make sure that policies implying a deterministic transfer to the old is always feasible, no matter what the realizations of W_t and θ_t.

The linear production function specification is used to analyze the partial equilibrium welfare implications of debt and transfers through capital accumulation while keeping factor returns constant. The Cobb-Douglas function specification is used to analyze the general equilibrium welfare implications of debt and transfers through capital accumulation and changes in factor returns. The two specifications can lead to different policy implications.

Factor markets. Factor markets are perfectly competitive. The wage W_t and the rental rate of capital R_t equal their marginal productivity in equilibrium:

$$R_t = A_t F_K(K_{t-1}, L_t) \tag{7}$$

$$W_t = A_t F_L(K_{t-1}, L_t) \tag{8}$$

More specifically, in the Cobb-Douglas case:

$$R_t = \alpha A_t K_{t-1}^{\alpha-1} \tag{9}$$

$$W_t = (1-\alpha) A_t K_{t-1}^{\alpha} \tag{10}$$

In the linear case:

$$R_t = \alpha A_t \tag{11}$$

$$W_t = (1-\alpha) A_t \tag{12}$$

2.3 Capital Motion

The investment of the *young* in period t adds to the capital stock that is used to produce output on period $t+1$ in combination with the labor supplied by the *young* generation of period $t+1$. The capital motion equation is $K_t = I_t + (1-\delta)K_{t-1}$ where δ is the depreciation rate.

Depreciation rate. Assume that capital fully depreciates after one period (i.e after 25 years): $\delta = 1$. Thus:

$$K_t = I_t \tag{13}$$

2.4 Government

The government can implement inter-generational transfers and set T_t accordingly every period. It can also decide to issue an initial amount of risk-free debt D_0 and rollover debt D_t at the real interest rate R_t^f every period. If the government starts a debt rollover policy, then the supply of government debt is inelastic. Put differently, the government issues every period the exact amount of debt necessary to repay the debt maturing in this period. As the model is a closed economy, market clearing requires that *young* households hold all the debt inelastically supplied by the government. One implication is that *young* households are constrained in their quantity of safe asset holdings, but their demand function is used to determine the equilibrium rate of return they require for holding a given quantity D_t. Absent default, the government's debt dynamics is governed by:

$$D_t = R_{t-1}^f D_{t-1} \tag{14}$$

Default. If the debt rollover fails, the government taxes the *young* so as to bring the debt back to its target value D^*. This strong assumption ensures that debt is perceived as perfectly safe by households. Failure is defined as the point where debt goes above its exogenous upper limit $\bar{D}$. Thus the tax is: $\theta_t = D_t - D^*$ if $D_t > \bar{D}$ and $\theta_t = 0$ otherwise.

2.5 Households decisions

***Old* households**. Households have no bequest motive and therefore do not accumulate assets in the last period of their life. Thus, *old* households consume all their income.

***Young* households**. Since labor supply is exogenous, *young* household decisions can be described by its asset demand functions, which then determine consumption through the household budget constraint. There are two assets in the economy: physical capital and risk-free debt.

The *young* households' maximization problem can be rewritten:

$$\max_{I_t, D_t} \mathbb{U} = (1-\beta)\ \log(W_t + X - T_t - I_t - D_t - \theta_t) + \frac{\beta}{1-\gamma}\ \log(\mathbb{E}_t[(R_{t+1} I_t + R_t^f D_t + T_{t+1})^{1-\gamma}]) \tag{15}$$

The first order condition with respect to I_t is:

$$\frac{1-\beta}{(W_t + X - T_t - I_t - D_t - \theta_t)} = \beta \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t + R_t^f D_t + T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t + R_t^f D_t + T_{t+1})^{1-\gamma}]} \tag{16}$$

This equation is used to derive the optimal demand for savings in physical capital. Similarly, the first order condition with respect to D_t is:

$$\frac{1-\beta}{(W_t + X - T_t - I_t - D_t - \theta_t)} = \beta \frac{R_t^f \mathbb{E}_t[(R_{t+1}I_t + R_t^f D_t + T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t + R_t^f D_t + T_{t+1})^{1-\gamma}]} \tag{17}$$

Market clearing in the economy requires that the *young* hold all debt issued by the government. At the margin, households are indifferent between holding safe debt and risky capital. This condition is used to pin down the interest rate on government debt. Put differently, the *young* are constrained in their quantity of safe asset holdings, but their demand function is used to determine the equilibrium rate of return they require on this asset given the quantity D_t. Using the household first order conditions derived above, and noting that the right hand side is equal in both equations (16) and (17), the safe interest rate, which is determined by the equilibrium of inelastic supply from the government and the demand from the *young*, must satisfy:

$$R_t^f = \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t + R_t^f D_t + T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t + R_t^f D_t + T_{t+1})^{-\gamma}]} \tag{18}$$

If there is debt in the economy ($D_t > 0$), then the investment in physical capital I_t and the risk-free rate R_t^f consistent with the *young* demand function evaluated at D_t are obtained by solving simultaneously equations (16) and (18) derived from the first order conditions every period. If there is no debt in the economy $D_t = 0\ \forall t$ then only equation (16) is used to obtain the optimal savings in physical capital I_t, and equation (18) defines the shadow risk-free rate.

3 Inter-generational Transfers and Long-Term Welfare Implications

This section studies the steady-state effects of inter-generational transfers on capital accumulation and welfare. Assume the economy is initially in a steady-state without government intervention. This steady-state is characterized by a combination of an average value for the return on capital and an average value for the risk-free interest rate. This section analyzes the change in steady-state welfare following the introduction of a inter-generational transfer policy for multiple combinations of average rates without government intervention. More specifically, this section analyzes 3 types of government policies: a pay-as-you-go (PAYGO) system with fixed transfers, a PAYGO system with stochastic transfers, and a wage subsidy financed by *old* households.

3.1 Calibration

Absent government intervention (i.e. $D_t = T_t = 0\ \forall t$), households receive an income when *young*. They consume part of it, and save the rest in risky capital. Capital earns a stochastic return next period, and *old* households consume all their proceeds. As shown in the appendix, the model can be calibrated for different values of the average shadow risk-free and average risky interest rates. Each combination of average rates represents a steady-state without government intervention. In the linear (respectively Cobb-Douglas) case, the parameters μ (respectively β) and γ are chosen so as to fit a set of pairs of values for the average safe rate and the average risky rate. The calibration of parameters follows Blanchard (2019). This section considers net annual average risky rates between 0% and 4%, and net annual risk-free rates between -2% and 1%. The following coefficients are chosen a priori: $\alpha = \frac{1}{3}$ (the capital share), $\beta = 0.325$ in the linear case and $\mu = 3$ in the Cobb-Douglas case, and $\sigma = 0.2$. In all simulations, the initial fixed endowment is equal to 100 percent of the average wage absent government intervention ($X = W^*$).

3.2 PAYGO: Fixed Transfers

This subsection reproduces the government policy described in Blanchard (2019) as a benchmark. Assume initially this economy had neither public debt nor social security system and the government decides to generate inter-generational transfers. More specifically, the government introduces a PAYGO system with fixed transfers: the amount paid by the *young* is fixed (assuming this amount is lower than the minimum realization of their income so that the transfer can always be made) and thus the payment to the *old* is certain. Given that the wage follows a log-normal distribution and thus can be arbitrarily small, the fixed endowment received by the *young* is needed to make sure that the deterministic transfer to the old is always feasible, no matter what the realizations of W_t and θ_t. This subsection analyzes two policies:

Policy 1: $T_t = \tau I^* \ \forall t$, where I^* is the initial steady-state investment absent government intervention. Calibration: $\tau = 5\%$. The government taxes the *young* a fixed amount and give it to the *old* as pensions. Here, as there is no growth in the economy, the gross return on this kind of savings is certain and equal to 1. The transfer is calibrated to 5% of steady-state savings absent government intervention.

Policy 2: $T_t = \tau I^* \ \forall t$. Calibration: $\tau = 20\%$. Same as Policy 1, but with a transfer equal to 20% of steady-state savings absent government intervention.

Figures 1a and 2a report the results from the simulations with the linear production function. The dots represent the change in steady-state welfare following the implementation of a given policy, for different calibrations of the average return to capital and average risk-free rate which characterized the steady-state without government intervention. This specification focuses on the partial equilibrium impact of higher debt on welfare through capital accumulation. As can be seen visually, policies 1 and 2 are welfare improving as long as the risk-free rate is lower than the growth rate (here, normalized to 0). The average return to capital does not matter to a first order approximation. The size of the change in welfare increases with the level of debt. If policy 1 leads to an increase (respectively decrease) in welfare, then policy 2 leads to a higher increase (respectively decrease) in welfare. Intuitively, in the initial steady-state without government, *young* households were indifferent between investing in the risky asset or in the risk-free asset at the prevailing average rates. The policy offers a safe gross return of 1 (i.e a net return of 0%). If the prevailing safe rate was below the growth rate (i.e negative), then the policy offers agents a higher safe return than the one which was leaving them indifferent at the margin between investing in the risky or the risk-free asset.

In general equilibrium, the introduction of the policy leads to a decrease in private investment, and thus to a decrease in wage-income and an increase in the marginal product of capital. These effects increase the safe interest rate that leaves agents indifferent between investing at the margin in risky capital or in the risk-free asset. If the introduction of the policy implies that this safe interest rate becomes positive at some point, then the policy is welfare decreasing. Put another way, the policy offers a safe return of 0% while the safe return that would actually make agents indifferent at the margin would have to be positive. Figures 1b and 2b report the results from the simulations with the Cobb-Douglas production function. As explained above, both interest rates matter now. A lower average risky rate and a lower average risk-free rate are associated with a higher welfare gain (or, lower welfare decrease). Policy 1 is welfare improving for a risk-free rate 2% below the growth rate as long as the risky rate is less than 2% above the growth rate. It is also welfare improving for a risk-free rate 1.5% and 1% below the growth rate as long as the risky rate is, respectively, less than 1.75% and 1% above the growth rate. The trade-off becomes less attractive as the debt increases. Policy 2 is welfare improving for a risk-free rate 2% below the growth rate as long as the risky rate is less than 1.5% above the growth rate. For an initial average annual risky rate of 2% and an initial average annual risk-free rate of -1%, policy 2 would lead to a 1.5% decrease in steady-state welfare. If we consider the production function to be Cobb-Douglas in the long-run then the set of average risk-free and risky rates that would lead to an increase in steady-state welfare is limited.

3.3 PAYGO: Stochastic Transfers

So far the analysis replicated Blanchard (2019). Assume instead that the government introduces a PAYGO system with stochastic transfers: the tax rate on wages is fixed, i.e. the total amount paid by the *young* is stochastic, and thus the payment to the *old* is uncertain. This subsection shows that a policy introducing stochastic transfers is welfare decreasing in the long-run if the average return to capital is higher than the growth rate. A formal proof for the linear production function specification is provided in the appendix. The two following policies are analyzed:

Policy 3: $T_t = \tau W_t \ \forall t$. Calibration: $\tau = 5\%$. The government taxes the *young* a share of their wage and give it to the *old* as pensions. As there is no growth in the economy, the average expected gross return on this kind of savings is equal to 1. However, because wages are stochastic every period, the return on this kind of savings is uncertain. The transfer is calibrated to 5% of the current wage rate.

Policy 4: $T_t = \tau W_t \; \forall t$. Calibration: $\tau = 20\%$. Same as Policy 3, but with a tax rate equal to 20%.

Figures 3a and 4a report the results from the simulations with the linear production function. As can be seen visually from figures 3 and 4, the welfare benefits of policies 3 and 4 only depend on the average return to capital. Policies 3 and 4 are welfare decreasing as long as the risky rate is higher than the growth rate (here, normalized to 0). The average risk-free rate does not matter to a first order approximation. The size of the decrease in welfare increases with the level of debt. Intuitively, in the initial steady-state without government, *young* households were indifferent between investing risky capital or in the risk-free asset at the prevailing average rates. The policy offers an average risky gross return of 1 (i.e a net return of 0%). If the prevailing risky rate was above the growth rate (i.e positive), then the policy offers agents a lower risky return than the one which was leaving them indifferent at the margin between investing in the risky or the risk-free asset. Note that there is a symmetry in the degree of uncertainty facing labor and capital returns in the model. Thus, there is no benefit associated with a less risky portfolio by substituting private savings for public savings: the transfer scheme gives agents an asset with the same uncertainty, but a different mean, from what they can already do, invest in capital. Then, the long-term desirability of the introduction of such transfer scheme boils down to a comparison of the average expected rate of return of private savings in the risky asset (the marginal product of capital) to the average expected rate of return of such transfer (equal to the growth rate of the economy, here zero). This is the partial equilibrium effect.

Figures 3b and 4b report the results from the simulations with the Cobb-Douglas production function. This specification focuses on the general equilibrium impact of higher debt on welfare through capital accumulation and change in prices. While the introduction of policies 3 and 4 leads to a lower capital level, and thus a higher marginal product of capital and lower wage, their welfare analysis is very similar to the one in partial equilibrium: the condition depends on the expected marginal product of capital and not on the risk-free interest rate. However, because the introduction of such policies leads to a lower capital accumulation, lower wages, and a higher marginal product of capital, the threshold on the average risky rate such that the policy is welfare improving is now lower (it was equal to 0% in the linear production specification). The bigger the transfer, the lower the threshold on average return to capital. Intuitively, the transfer offers an average risky return of 0% while the risky return that would actually leave agents indifferent at the margin would have to be even higher than absent government intervention due to the crowding-out effect on factor returns. In particular, even for an average risky rate of 0%, policy 3 is now welfare decreasing. Policy 4 decreases long-run welfare as long as the average risky interest rate is less than 0.5% below the growth rate. For an average risky rate of 0%, the introduction policy 4 would imply a decrease in steady-state welfare of about 2%. Thus, in the current environment, the introduction of such policies is likely to be welfare decreasing in the long-run.

3.4 Wage Subsidies

Building on the previous analysis, this subsection argues that a wage subsidy can be welfare improving in the long-run. The formal proof for the linear production function specification is identical to the one for a PAYGO with stochastic transfers, the only difference being that τ is negative here while it was positive in the previous subsection. This subsection discusses the two following policies:

Policy 5: $T_t = \tau W_t \ \forall t$. Calibration: $\tau = -5\%$. The government finances a wage subsidy to the *young*. The subsidy represents a share of their wage and is financed by taxing the *old*. As there is no growth in the economy, the average tax repayment is equal to the average subsidy. However, because wages are stochastic every period, the tax is uncertain. The subsidy is calibrated to 5% of the current wage rate.

Policy 6: $T_t = \tau W_t \ \forall t$. Calibration: $\tau = -20\%$. Same as Policy 5, but with a subsidy rate equal to 20%.

Figures 5a and 6a report the results from the simulations with the linear production function. Policies 5 and 6 are welfare improving as long as the risky rate is higher than the growth rate (here, normalized to 0). The average risk-free rate does not matter to a first order approximation. The size of the increase in welfare increases with the level of debt. Intuitively, such policy gives an extra income to *young* households. As there is no growth in this economy, the average expected repayment is equal to the average subsidy. This extra income can be understood as a loan with an average expected net interest rate of 0%, and can be invested in the risky asset. If the prevailing risky rate is above the growth rate (i.e positive), then the policy increases steady-state welfare as it offers agents the possibility to invest some extra income at an average expected return higher than the average expected cost of this extra income. Additionally, as there is a symmetry in the degree of uncertainty facing labor and capital returns in the model, the policy offers agents an additional benefit from income diversification and lower risk in their portfolio. Indeed, if the return on capital is high, then *old* households would have to repay more for the wage subsidy, while they would repay less if the return on capital is low. Thus, the wage subsidy allows agents to better diversify away their risk[3]. This is the partial equilibrium effect.

Figures 5b and 6b report the results from the simulations with the Cobb-Douglas production function. While the introduction of policies 5 and 6 leads to a higher capital level, and thus a higher wage and lower marginal product of capital, their welfare analysis is very similar to the one in partial equilibrium: the condition depends on the expected marginal product of capital and not on the risk-free interest rate. However, the threshold on the average risky rate such that the policy is welfare improving is now higher (it was equal to 0% in the linear production specification). The bigger the transfer, the higher the threshold on average return to capital. As long as the economy remains *dynamically efficient*, the policy is unambiguously welfare improving in the long-run. If the wage subsidy leads to a capital accumulation that drives down the average expected (net) return to capital below 0, i.e below the average expected net interest repayment on the subsidy (or equivalently to the steady-state growth rate of the economy), the policy would decrease steady-state welfare. Intuitively, as the stock of capital increases, the risky return that would actually leave agents indifferent at the margin would be lower than absent government intervention due to the crowding-out effect on factor returns. In particular, policy 6 decreases long-run welfare as long as the average risky interest rate is less than 1% above

[3]The suboptimality of risk allocation in stochastic overlapping-generations models has been discussed in several papers, including Bohn (1998) and Shiller (1999).

the growth rate. For an initial average annual risky rate of 2% and an initial average annual risk-free rate of -1%, policies 5 and 6 would respectively lead to a 1.6% and 4.7% increase in steady-state welfare. Thus, in the current environment, the introduction of such policies is likely to be welfare increasing in the long-run. However, a subsidy has clearly a negative impact on the current old generation which pays but does not benefit from it. This section has omitted from transitional effects, which are discussed in the next section.

4 Debt Rollovers, Transfers and Short-Term Welfare Implications

The previous section discussed the steady-state welfare implications of transfers. However, there are important transitional effects, and the impact varies across generations. Thus, this section studies the transitional welfare implications of debt rollovers and transfers. The main finding is that the economy is *dynamically inefficient* in the current low rate environment: the simulations in subsection 4.2 and 4.3 show that a combination of either fixed transfers or a debt rollover policy and a wage subsidy could generate a Pareto welfare improvement.

4.1 Calibration

For all analyzed policies, I run 1,000 paths of the economy with and without intervention. In this section, the model is calibrated for one specification: the values for γ and β (respectively μ) in the Cobb-Douglas (respectively linear) specification which correspond to $\mathbb{E}\, R = 2\%$ and $\mathbb{E}\, R^f = -1\%$ in steady-state absent government intervention. This makes the results comparable to Blanchard (2019). As before, the initial fixed endowment is equal to 100 percent of the average wage absent government intervention in all simulations. The following parameters are chosen a priori: $\bar{D} = 0.1725 I^*$ and $D^* = 0.4 D_0$, where I^* is the steady-state investment absent government intervention, and D_0 is the initial debt issuance. The next subsection shows that such parameters give results very similar to Blanchard (2019). A higher upper debt limit $\bar{D}$ would make default less likely for early generations, but would, *ceteris paribus*, increase the cost on generations hit by default. A higher target debt level D^* would decrease the cost on generations hit by default, but would, *ceteris paribus*, make default more likely. The results are robust to different values for the default parameters.

4.2 Debt Rollovers

This subsection reproduces Blanchard (2019) as a benchmark. The government initially issues debt D_0 and distributes the proceeds as transfers to the current *old* generation, does not raise taxes (as long as the debt rollover does not fail) and let debt dynamics play out. More specifically, the policy is:

Policy 7: $T_t = 0 \; \forall t$; $D_0 = \kappa I^*$. Calibration: $\kappa = 15\%$. The initial debt (gift to the *old*) is calibrated to 15% of steady-state investment absent government intervention.

Figures 7.1a and 7.2a show that, in the linear case, debt rollovers typically do not fail and welfare is increased throughout. For the generation receiving the initial transfer associated with debt issuance, the effect is clearly positive and large (black dot). For later generations, while they are, at the margin, indifferent between holding safe debt or risky capital, the infra-marginal gains (from a less risky portfolio) imply slightly larger utility. But the welfare gain is small (equal initially to about 0.28 percent and decreasing over time), compared to the initial welfare effect on the old from the initial transfer (8.75 percent).

Figures 7.1b and 7.2b show that, in the Cobb-Douglas case, with the same values of average rates absent debt, bad shocks, which lead to higher debt and lower capital accumulation, lead to increases in the risky rate, and by implication, larger increases in the safe rate. While welfare still goes up for the first *young* generation (by about 2 percent), it is typically negative thereafter. In the case of successful debt rollovers, the average adverse welfare cost decreases as debt decreases over time. In the case of unsuccessful rollovers, the adjustment implies a larger welfare loss when it happens.

4.3 Transfers and Subsides

Results from the previous subsection show that if the production function is Cobb-Douglas then current generations benefit from the debt rollover at the expense of future generations. This does not suggest that the economy could be *dynamically inefficient* as some generations are made better off at the expense of future generations. This subsection shows that a well designed policy can actually lead to a Pareto improvement.

As discussed in the previous section, the introduction of fixed transfers benefits early generations at the expense of future generations. Conversely, a wage subsidy benefits future generations at the expense of early generations (in particular the current *old* generation). This subsection asks the following question:

Could a social planner increase welfare for all generations by combining both policies?[4] The following policy is analyzed:

Policy 8: $T_t = \tau_I I^* + \tau_W W_t \; \forall t$. Calibration: $\tau_I = 20\%$ and $\tau_W = -\frac{\tau_I I^*}{W^*}\%$, where I^* and W^* are the initial steady-state investment and wage absent government intervention. The initial transfer (gift to the *old*) is equal to 20% of steady-state investment absent government intervention. The subsidy rate is set such that the initial payment made by the *old* to the *young* exactly offsets the utility they obtain from the initial debt increase. Put differently, the welfare of current *old* households is left unchanged.

Figures 8.1a and 8.2a show that, in the linear case, the current *old* generation (black dot) is indifferent to the policy while welfare goes up for later generations (by about 3 percent on average). In all 1,000 simulations welfare is increased for all generations. This policy constitutes a Pareto welfare improvement.

Figures 8.1b and 8.2b show that, in the Cobb-Douglas case, welfare is also increased throughout. The current *old* generation is indifferent to the policy while welfare goes up for the current *young* generation (by about 2.6 percent), whose wage was set before the introduction of the policy and thus is not negatively affected by it. Investment goes down and thus wages decrease, due to the crowding out effect coming from transfers. At the same time, the wage subsidy fosters investment and consumption (as shown in the proof in the appendix), thus limiting the adverse price effect. The resulting outcome of the 2 opposite effects is such that welfare is increased for all generations in all simulations. In both cases, there is lower investment, and thus lower capital accumulation. The policy leads to a Pareto welfare improvement and a decrease in the steady-state level of capital.

[4] DeLong and Waldmann (2019) have recently published a blog post which also argues that a debt rollover combined with a wage subsidy can generate a Pareto welfare improvement. However, their sequence of wage subsidies requires an 'extremely arithmetically-inclined' state according to their own words, while I assume the government policy takes the form of a simple *ex ante* announcement.

4.4 Debt Rollovers and Subsidies

This subsection extends the previous analysis by focusing on debt rollovers instead of fixed transfers. Compared to transfers, debt rollovers offer a rate of return R_f, which is typically lower than the gross rate of return on fixed transfers, i.e. 1. However, because R_f is typically lower than 1, the debt level, and thus the negative price effect arising from crowding out of capital, vanishes over time. The following policies are analyzed:

Policy 9: $T_t = \tau W_t \ \forall t$; $D_0 = \kappa I^*$. Calibration: $\kappa = 10\%$; $\tau = -\frac{D_0}{W^*}$, where W^* is the initial steady-state wage absent government intervention. The initial debt (gift to the *old*) is equal to 10% of steady-state investment absent government intervention. The subsidy rate is set such that the initial payment made by the *old* to the *young* exactly offsets the utility they obtain from the initial debt increase. Put differently, the welfare of current *old* households is left unchanged.

Figures 9.1a and 9.2a show that, in the linear case, debt rollovers typically do not fail and average welfare is increased throughout, although in a few simulations welfare decreases for some generations. The current *old* generation is indifferent to the policy while welfare goes up for later generations (by about 1.5 percent on average). In a few simulations some generations experience a decrease in welfare, but the decrease is small.

Figures 9.1b and 9.2b show that, in the Cobb-Douglas case, debt rollovers typically do not fail and welfare is increased throughout, in all simulations. The current *old* generation is indifferent to the policy while welfare goes up for the current *young* generation (by about 1 percent), whose wage was set before the introduction of the policy and thus is not negatively affected by it. Note that this first generation does not contribute on net to the system when they are *young*, but benefit from it when they are *old*, which explain the relatively high welfare gain, especially when compared to the following generation. Initially, investment goes down and thus wages decrease, due to the crowding out effect coming from the debt rollover. At the same time, the wage subsidy fosters investment and consumption (as shown in the proof in the appendix), thus limiting the adverse price effect. The resulting outcome of the 2 opposite effects is such that welfare is increased for generations 1 and 2 in all simulations, despite a lower wage than would otherwise prevail absent government intervention. It should be noted that generations 1 and 2 experience the lowest average increase in welfare as the economy transitions to lower debt. This is because agents are forced to invest an important share of their income in government debt, which pays a low safe interest rate, in order to rollover the debt. After generation 2, the stock of debt becomes relatively low, investment increases, the level of capital recovers and goes above the level it would have had absent government intervention. The negative effect of higher debt fades away as debt decreases and the wage subsidy effect on investment dominates: there is a *crowding in* effect. As a result, wages increase, and welfare is permanently increased for later generations. An economy in such low rate environment was initially *dynamically inefficient* as an appropriate combination of policies leads to a Pareto welfare improvement for all generations in all simulations. Compared to the previous policy, this policy leads to an increase in the steady-state level of capital. Put differently, such an economy was *dynamically inefficient* in the sense that there exists a combination of policies that leads to a Pareto welfare improvement. Yet, this economy was not necessarily over-accumulating capital. Indeed, policy 9, which combines a debt rollover and a carefully calibrated wage subsidy, leads to both a Pareto welfare improvement and a higher steady-state level of capital and output. What would be the optimal increase in initial debt D_0? A higher initial debt level makes the negative impact on investment and wages more persistent in the Cobb-Douglas specification. The lower the initial average safe rate, the faster this negative effect fades away. The lower the average risky rate, the lower the welfare cost of lower capital accumulation. At the same time, a higher initial debt level allows for a higher wage subsidy while leaving the current *old* generation indifferent to the policy. The higher the average risky rate, the higher the long-term welfare gains from the wage subsidy. Thus, while a lower average safe rate calls for a higher initial debt level, the impact of the average risky rate on the initial debt level is unclear. This is left for further research.

Finally, this subsection argues that an extended debt rollover —a policy that consists in issuing an additional amount of new debt every period and rolling over the entire stock of debt— could also lead to a Pareto welfare improvement if combined with a wage subsidy. The following policy is analyzed:

Policy 10: $T_t = \tau W_t \ \forall t$; $D_0 = \kappa I^*$; $D_{t+1} = R_t^f D_t + \varkappa D_0$. Calibration: $\kappa = 10\%$; $\varkappa = 7.5\%$; $\tau = -\frac{D_0}{W^*}$, where W^* is the initial steady-state wage absent government intervention.

Policy 10 is a combination of an extended debt rollover policy and a wage subsidy policy. The government announces *ex ante* that it will issue D_0 at $t = 0$, rollover this debt, and issue an additional amount $\varkappa D_0$ at every period. Put differently, every *young* agent knows that when *old* she will receive the returns from her forced savings in public debt plus an additional fixed amount, which will be raised from the *young* agents at that time. As shown in figure 10.1a, all generations gain from this policy in all simulations in the linear production function specification[5]. Policy 10 makes the debt level more persistent than policy 9 and thus is very similar to policy 8 (fixed transfer). However, compared to policy 8, policy 10 leads eventually to a higher investment level as debt decreases, although the convergence is much slower than with policy 9. Initially, investment goes down, but recovers progressively. Yet, wages do not change.

In the Cobb-Douglas production function specification, policy 10 increases welfare in all simulations for all generations. The analysis is similar to the linear production function case except that now wages initially go down and then gradually recover, although the convergence is much slower than with policy 9. Steady-state investment, capital, and wages are higher than absent government intervention. Under such calibration, the policy leads to a Pareto welfare improvement, regardless of the production function specification, while the steady-state level of capital is higher.

[5] More specifically, I run the simulations for 20 generations (500 years) and find that no generation experiences a decrease in welfare, in any simulation, compared to the counterfactual without government intervention.

5 Conclusion

This paper discusses the welfare implications of inter-generational transfers and debt rollovers in an economy where the growth rate is higher than the safe rate but lower than the average marginal product of capital. It draws three main conclusions.

First, if the policy intervention takes the form of a PAYGO system with stochastic transfers (i.e. a fixed tax rate on wages) then only the average risky rate matters, independently of the production function specification, to assess whether the policy is welfare improving or not in the long-run. In particular, in the linear production specification, the long-term welfare analysis boils down to a comparison of the average risky rate to the growth rate. This analysis, however, abstracts from transitional effects. Second, the simulations show that the economy is likely to be *dynamically inefficient* in such low rate environment as a social planner could generate a Pareto welfare improvement by starting a PAYGO system with fixed transfers while introducing at the same time inter-generational transfers taking the form of simply designed wage subsidies. Third, the combination of a debt rollover, or an extended debt rollover, and a wage subsidy could generate a Pareto welfare improvement, while leading to a higher level of steady-state capital. This challenges the view that *dynamic inefficient* stochastic OLG economies have over-accumulated capital.

The results suggest that there is a case for carefully designed debt policies in the current low rates environment. It would be interesting to understand how the optimal size in the initial debt increase varies with different combinations of the average safe and risky rates, or to include business cycle considerations as fiscal policy may take on greater importance in responding to future recessions in a low rates environment. This is left for further research.

References

Abel, Andrew B., N. Gregory Mankiw, Lawrence H. Summers, and Richard J. Zeckhauser (1989): "Assessing Dynamic Efficiency: Theory and Evidence." The Review of Economic Studies 56, no. 1: 1-19.

Ball, L., Elmendorf, D., and Mankiw, N. (1998): "The Deficit Gamble." Journal of Money, Credit and Banking, 30(4), 699-720.

Ball, Laurence and N. Gregory Mankiw (2007): "Intergenerational Risk Sharing in the Spirit of Arrow, Debreu, and Rawls, with Applications to Social Security Design," Journal of Political Economy, University of Chicago Press, vol. 115(4), pages 523-547, 08.

Blanchard, Olivier (2019): "Public Debt and Low Interest Rates." American Economic Review, 109 (4): 1197-1229.

Bohn, Henning (1998): "Risk Sharing in a Stochastic Overlapping Generations Economy," University of California at Santa Barbara, Economics Working Paper Series, Department of Economics, UC Santa Barbara.

Carroll, Christopher C. (2018): "Consumption with Optimal Portfolio Choice" Archive, Johns Hopkins University. Available at: http://www.econ2.jhu.edu/people/ccarroll/public/lecturenotes/AssetPricing/C-With-Optimal-Portfolio/

DeLong, Brad and Robert Waldmann (2019): "When Is an Economy Dynamically Inefficient?". Available at: https://www.bradford-delong.com/2019/04/dotting-is-and-crossing-ts-with-respect-to-olivier-blanchards-secular-stagnation-fiscal-policy-in-an-era-of-low-interest.html

Diamond, Peter A. (1965): "National Debt in a Neoclassical Growth Model," American Economic Review, 55, 1126–1150

Neil Mehrotra (2018): "Debt Sustainability in a Low Interest Rate World," 2018 Meeting Papers 285, Society for Economic Dynamics.

Merton, Robert C. (1969): “Lifetime Portfolio Selection under Uncertainty: The Continuous Time Case,” Review of Economics and Statistics, 50, 247–257.

Òscar Jordà, Moritz Schularick, and Alan M. Taylor (2017): “Macrofinancial History and the New Business Cycle Facts.” in NBER Macroeconomics Annual 2016, volume 31, edited by Martin Eichenbaum and Jonathan A. Parker. Chicago: University of Chicago Press.

Reiter, Michael (2015): “Solving OLG Models with Many Cohorts, Asset Choice and Large Shocks,” Economics Series, Institute for Advanced Studies.

Samuelson, Paul A. (1969): “Lifetime Portfolio Selection by Dynamic Stochastic Programming,” Review of Economics and Statistics, 51, 239–46.

Shiller, Robert J. (1999): ”Social security and institutions for intergenerational, intragenerational, and international risk-sharing,” Carnegie-Rochester Conference Series on Public Policy, Elsevier, vol. 50(1), pages 165-204, June.

Appendix - Proof

Assume for simplicity that $X = 0$.

With linear production function:

Factor returns are given by: $R_t = \alpha A_t$ and $W_t = (1-\alpha)A_t$. Note that:

$$\frac{1-\alpha}{\alpha} = \frac{W_t}{R_t} \tag{1}$$

Set $T_t = D_t = 0\ \forall t$. Without transfers, the maximization problem is:

$$\max_{I_t} \mathbb{U} = (1-\beta)\ \log(W_t - I_t) + \frac{\beta}{1-\gamma}\ \log(\mathbb{E}_t[(R_{t+1}I_t)^{1-\gamma}]) \tag{2}$$

The FOC simplifies to:

$$\frac{1-\beta}{(W_t - I_t)} = \beta \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t)^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t)^{1-\gamma}]} \tag{3}$$

After some algebra this leads to the following optimal investment decision:

$$I_t = \beta W_t \tag{4}$$

Now set $T_t = \tau W_t$, $D_t = 0\ \forall t$. With transfers, the maximization problem is:

$$\max_{I_t^L} \mathbb{U} = (1-\beta)\ \log(W_t(1-\tau) - I_t^L) + \frac{\beta}{1-\gamma}\ \log(\mathbb{E}_t[(R_{t+1}I_t^L + \tau W_{t+1})^{1-\gamma}]) \tag{5}$$

The FOC simplifies to:

$$\frac{1-\beta}{(W_t(1-\tau) - I_t^L)} = \beta \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t^L + \tau W_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t^L + \tau W_{t+1})^{1-\gamma}]} \tag{6}$$

After some algebra this leads to the following optimal investment decision:

$$I_t^L = \beta W_t(1-\tau) - \frac{1-\alpha}{\alpha}\tau(1-\beta) \tag{7}$$

Result 1. Assuming $0 < \alpha, \beta < 1$, it can be seen from (4) and (7) that $I_t^L > I_t$ if and only if $\tau < 0$. Conversely, $I_t^L < I_t$ if and only if $\tau > 0$.

Consumption when *young* and *old* absent government intervention are:

$$C_t^y = (1-\beta)W_t \tag{8}$$

$$C_{t+1}^o = R_{t+1} I_t \tag{9}$$

Consumption when *young* and *old* after the government intervention are:

$$C_t^{y,L} = W_t(1-\tau) - I_t^L = (1-\beta)[W_t(1-\tau) + \frac{1-\alpha}{\alpha}\tau] \tag{10}$$

$$C_{t+1}^{o,L} = R_{t+1} I_t^L + \tau W_{t+1} \tag{11}$$

By comparing (8) and (10), and using (1), it can be seen that $C_t^{y,L} \geq C_t^y$ if and only if $\tau \geq \tau R_t$. Similarly, by comparing (9) and (11), and using (1), it can be seen that $C_{t+1}^{o,L} \geq C_{t+1}^o$ if and only if $\tau \geq \tau R_t$.

Result 2. Consider two cases:

- If $R_t \geq 1$ then $C_t^{y,L} \geq C_t^y$ and $C_{t+1}^{o,L} \geq C_{t+1}^o$ if and only if $\tau < 0$.
- If $R_t \leq 1$ then $C_t^{y,L} \geq C_t^y$ and $C_{t+1}^{o,L} \geq C_{t+1}^o$ if and only if $\tau > 0$.

Summarizing the previous results, if $R_t \geq 1$ then $\tau < 0$ is welfare improving and $\tau > 0$ is welfare decreasing. Conversely, if $R_t \leq 1$ then $\tau > 0$ is welfare improving and $\tau > 0$ is welfare decreasing.

With Cobb-Douglas production function:

After some algebra this leads to the following optimal investment decision:

$$I_t^{CD} = \beta W_t(1-\tau)\frac{\alpha}{\alpha + \tau(1-\alpha)(1-\beta)} \tag{12}$$

Result 3. Assuming $0 < \alpha, \beta < 1$, it can be seen from (4) and (12) that $I_t^{CD} > I_t$ if and only if $\tau < 0$. Conversely, $I_t^{CD} < I_t$ if and only if $\tau > 0$. This effect is further amplified over time as higher (respectively lower) investment implies a higher (lower) wage.

Appendix - Figures

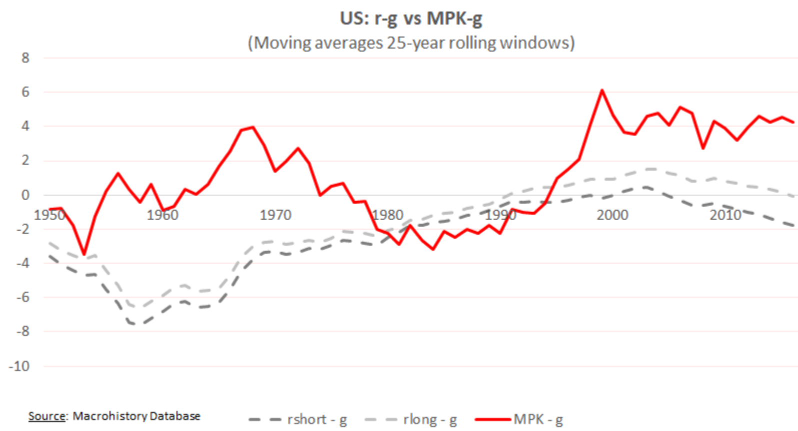


Figure A: Interest rates, stock returns and growth in the US

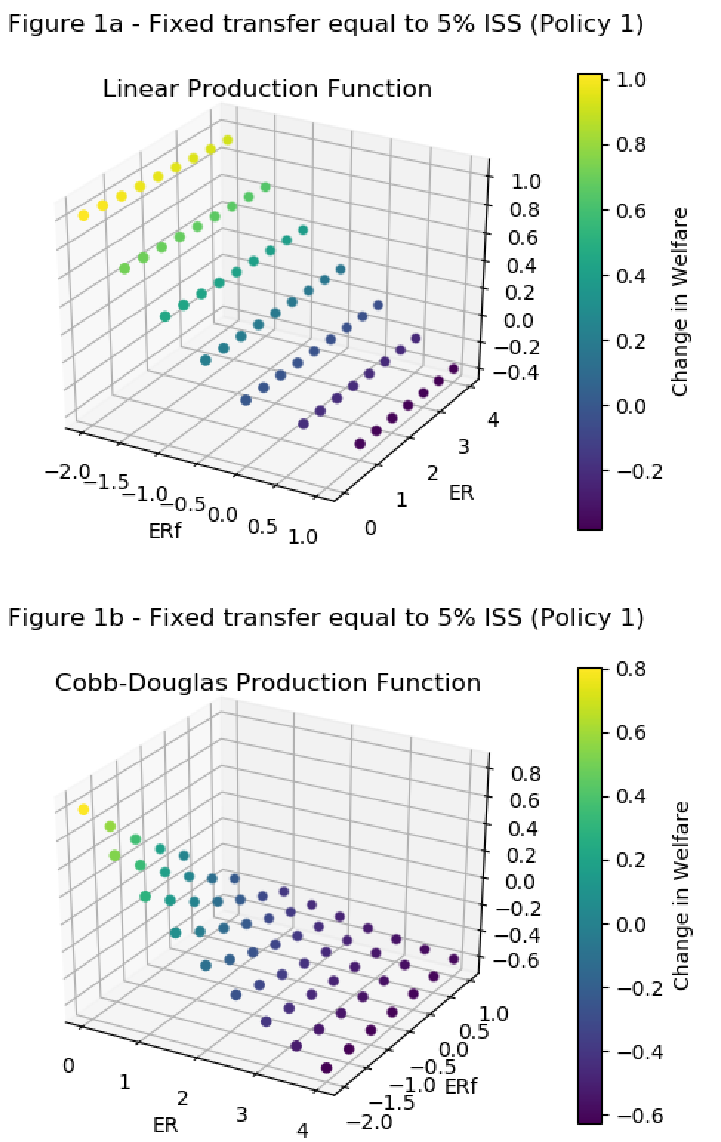
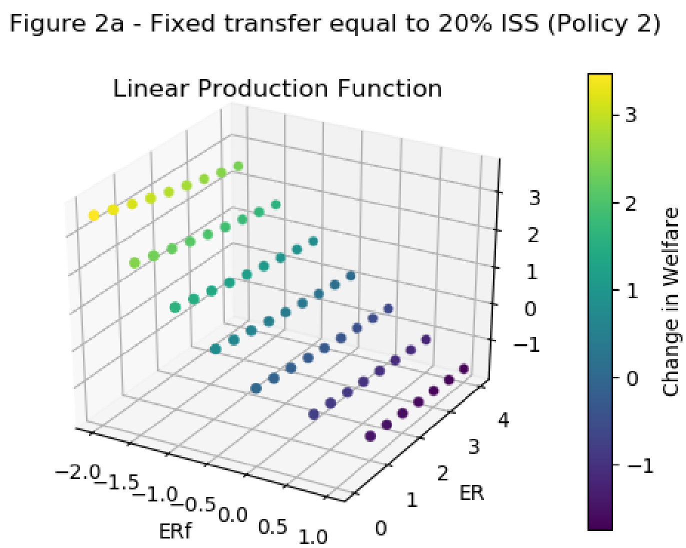


Figure 1: Long-term welfare implications of fixed transfers (Policy 1)



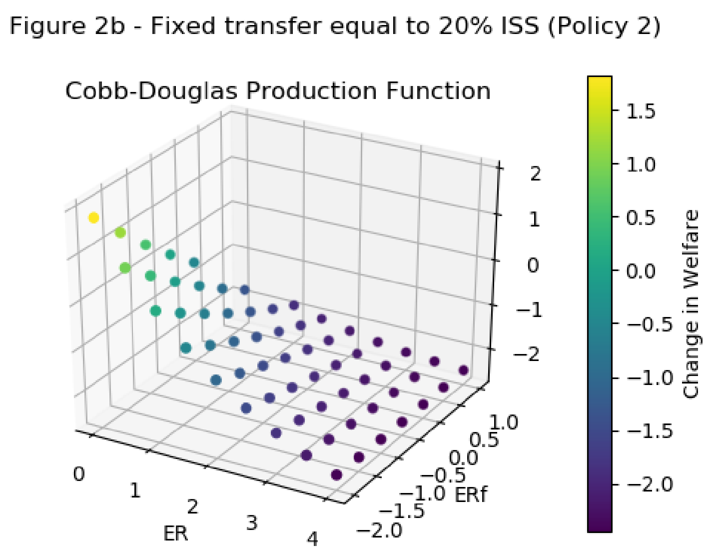
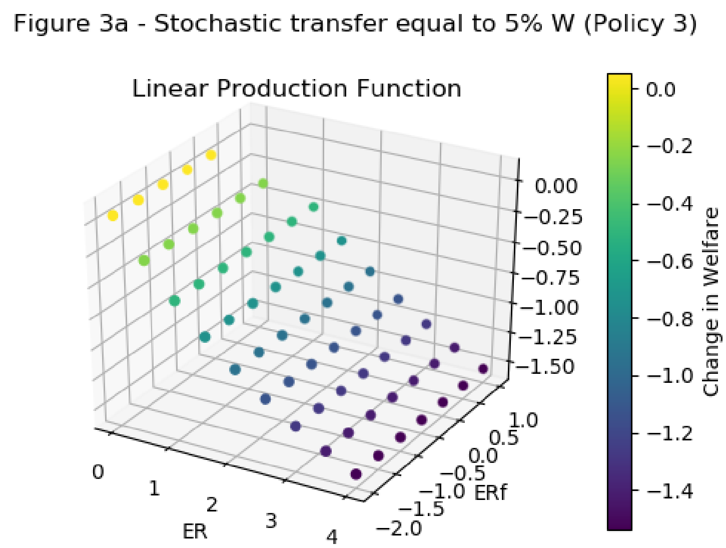


Figure 2: Long-term welfare implications of fixed transfers (Policy 2)



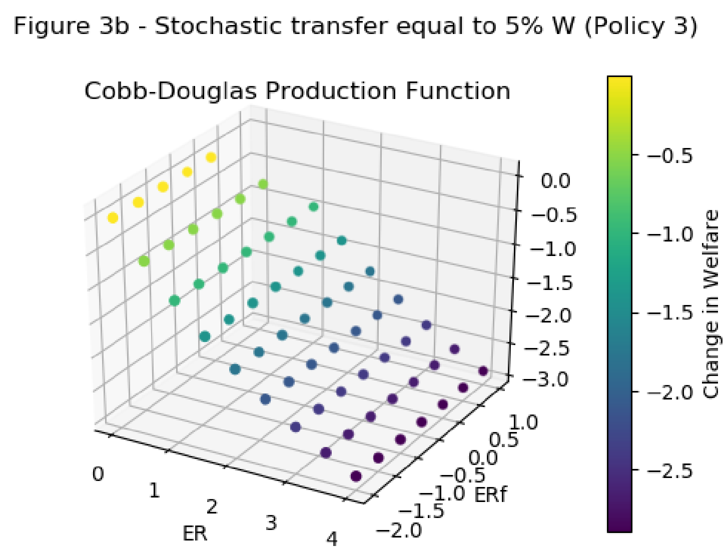


Figure 3: Long-term welfare implications of stochastic transfers (Policy 3)

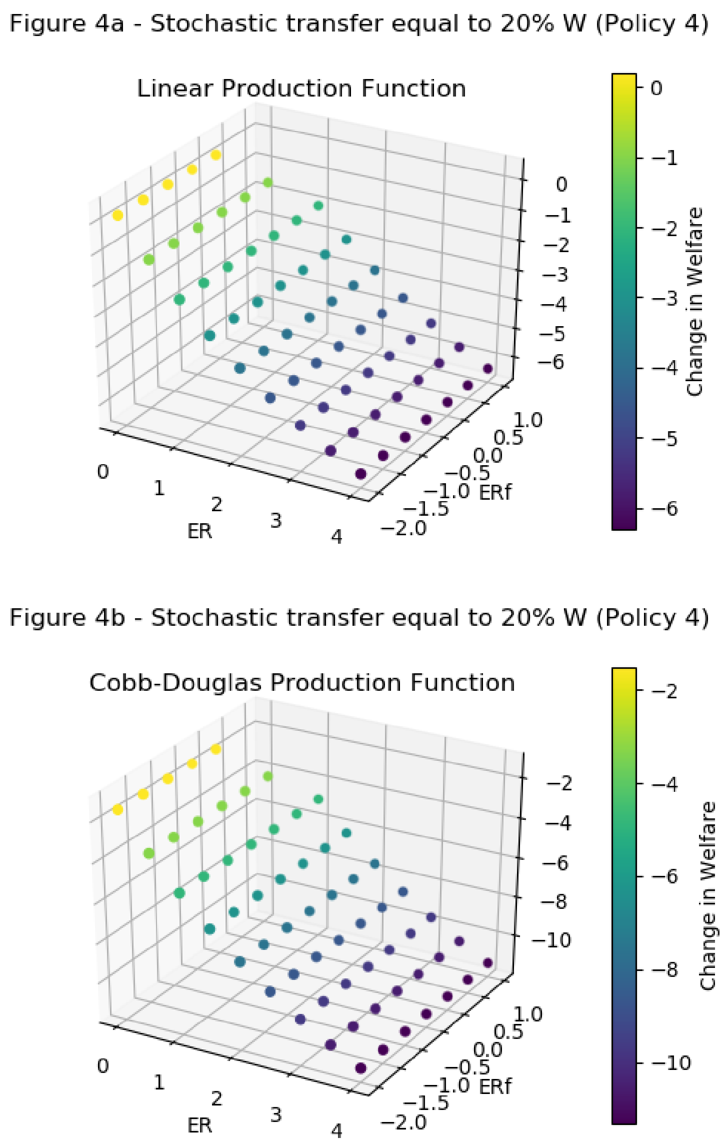


Figure 4: Long-term welfare implications of stochastic transfers (Policy 4)

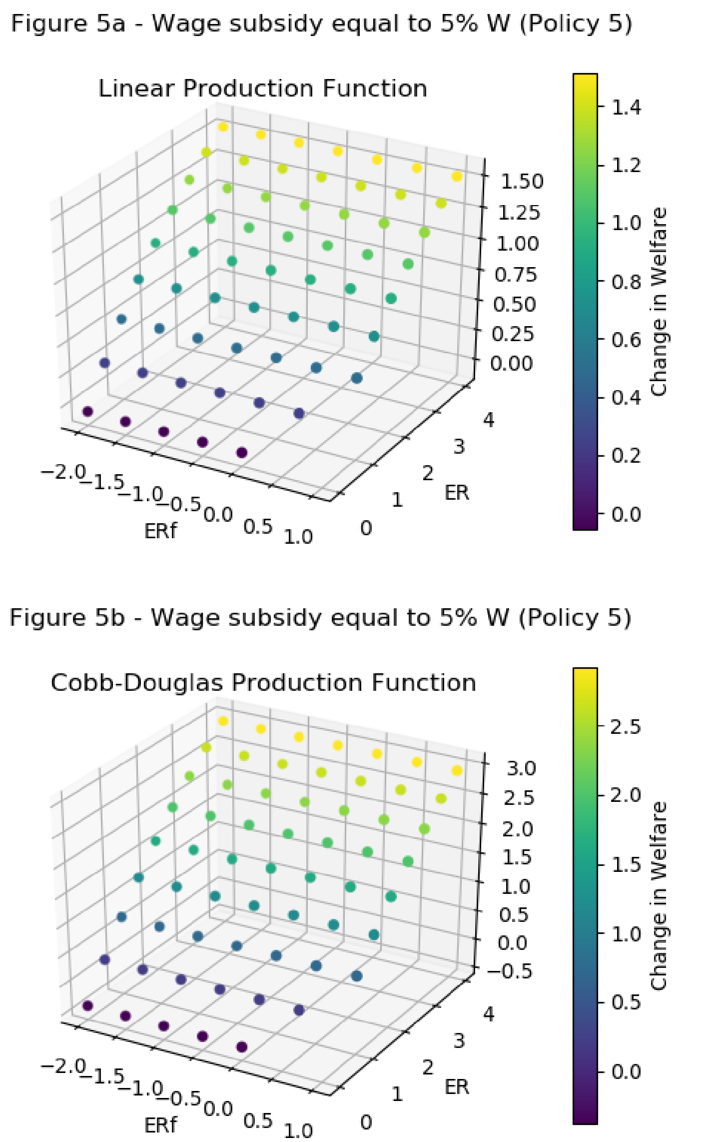
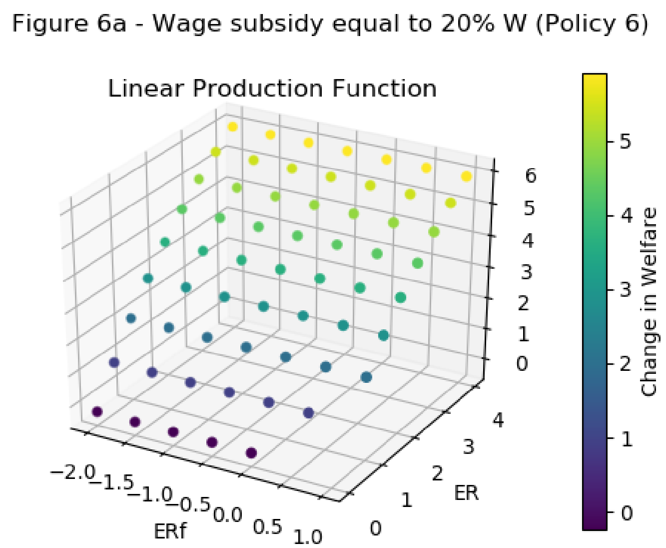


Figure 5: Long-term welfare implications of wage subsidies (Policy 5)



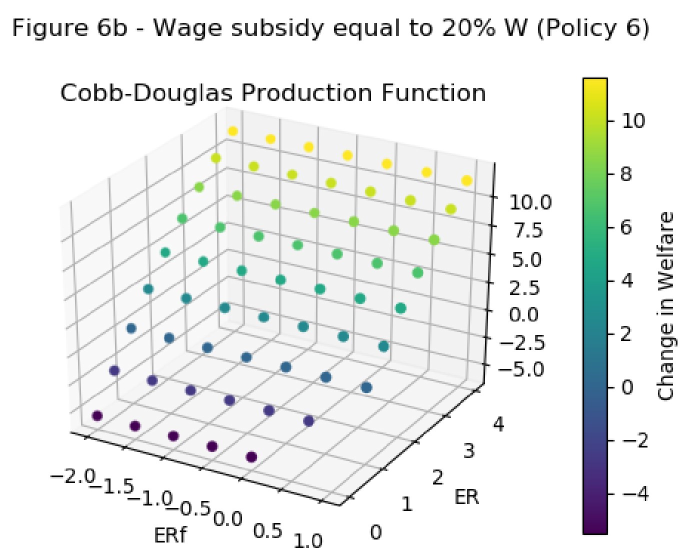


Figure 6: Long-term welfare implications of wage subsidies (Policy 6)

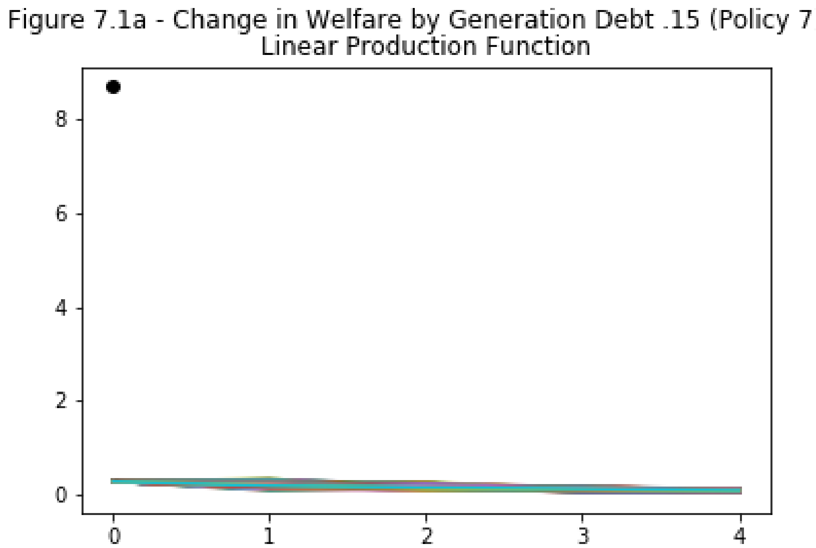


Figure 7.1a - Change in Welfare by Generation Debt .15 (Policy 7)
Linear Production Function
8
6
4
2
0
0
1
2
3
4

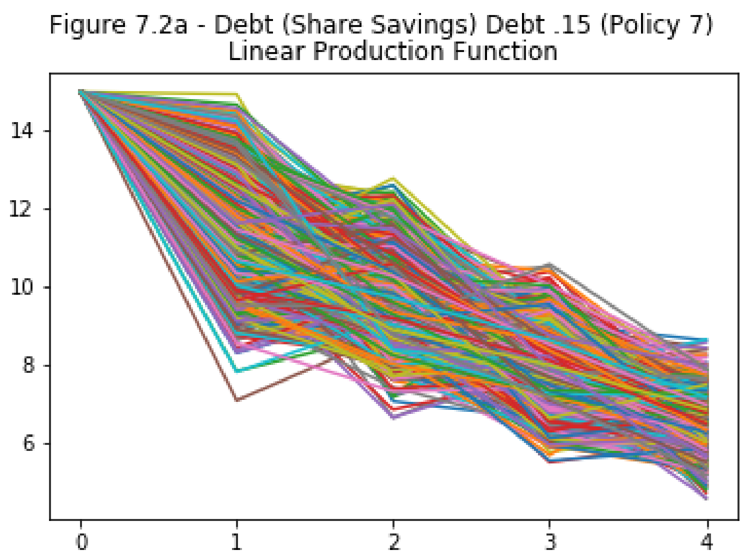


Figure 7.2a - Debt (Share Savings) Debt .15 (Policy 7)
Linear Production Function
14
12
10
8
6
0
1
2
3
4

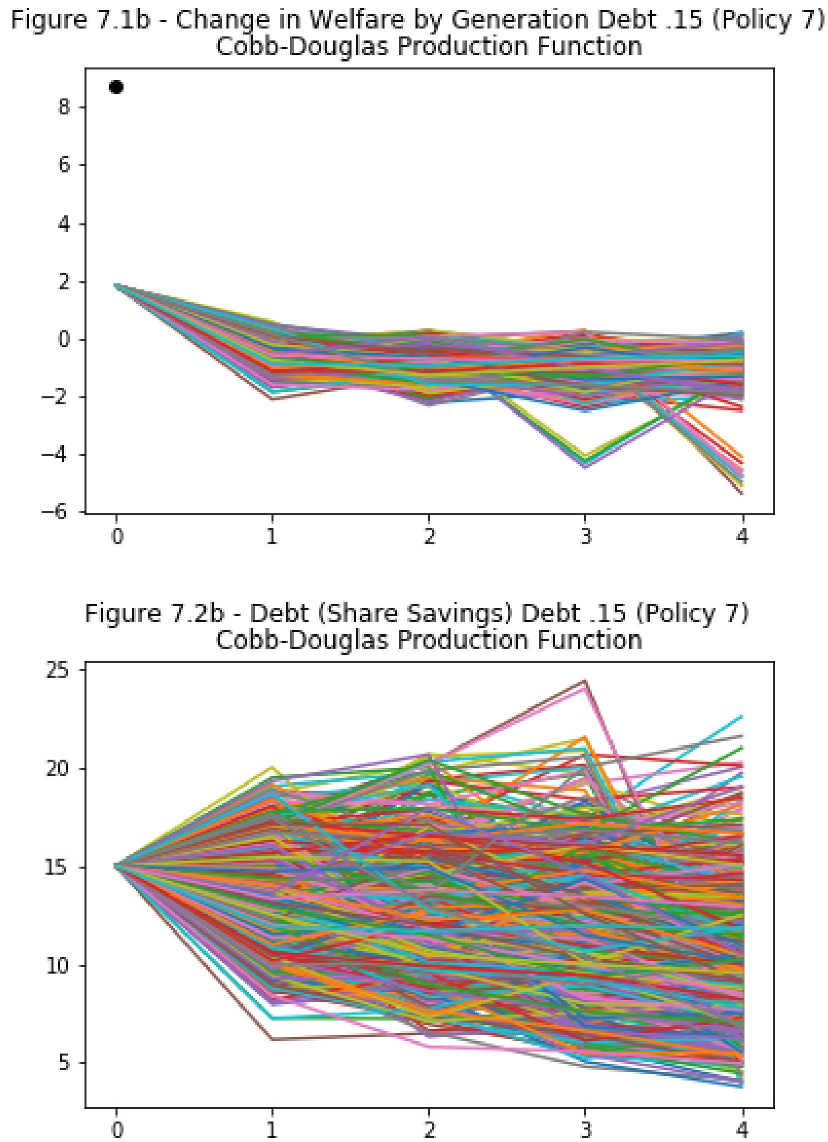


Figure 7: Short-term implications of debt rollovers (Policy 7)

jure 8.1a - Change in Welfare by Generation Transfers .20 (Tax) (Policy
Linear Production Function

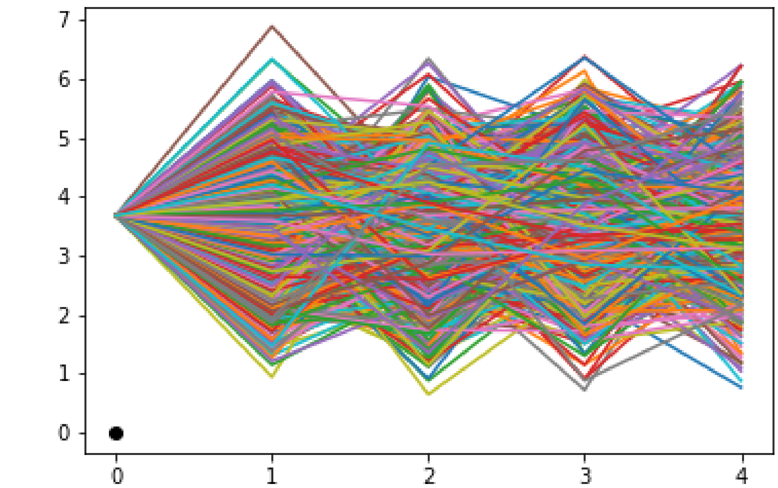
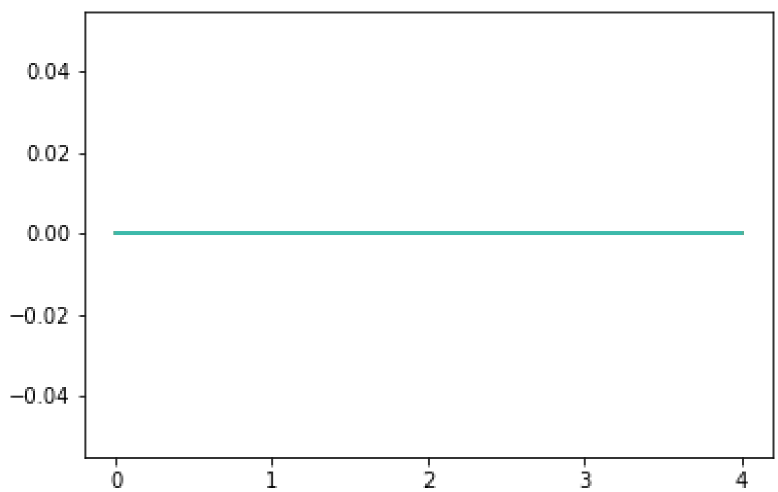
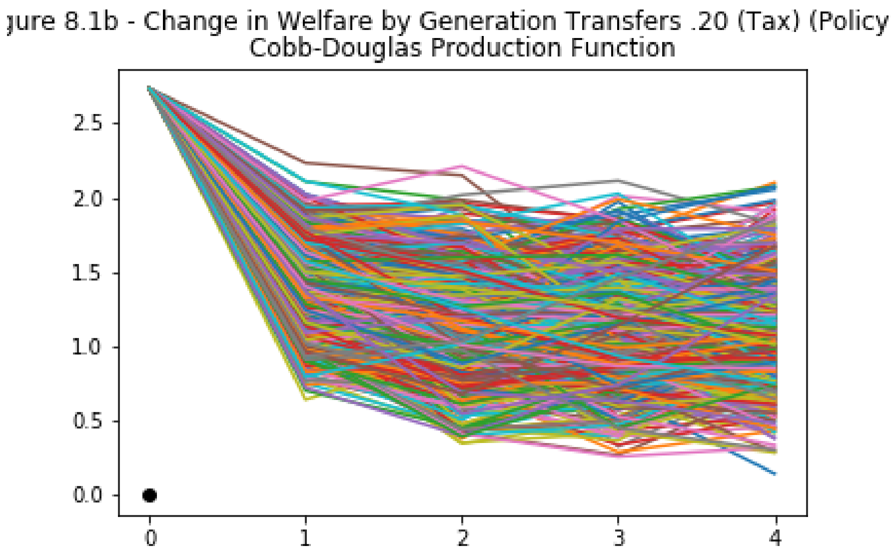


Figure 8.2a - Debt (Share Savings) Transfers .20 (Tax) (Policy 8)
Linear Production Function





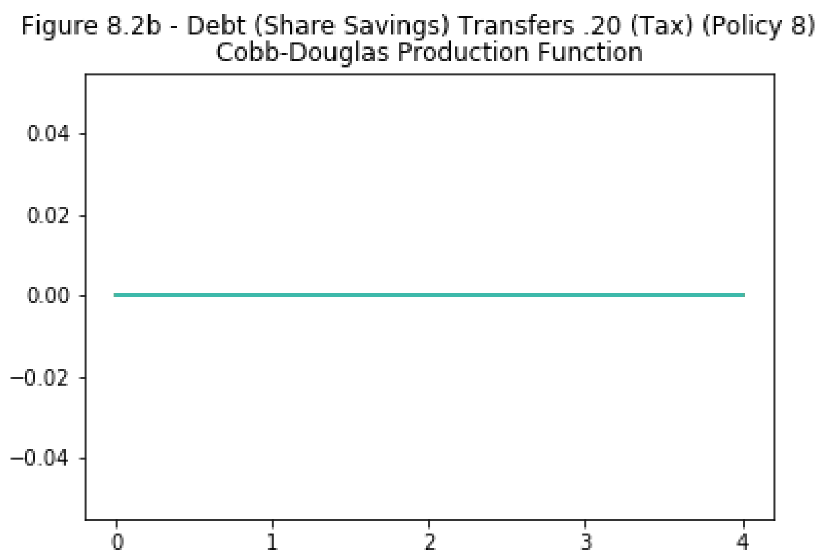


Figure 8: Short-term implications of fixed transfers and subsidies (Policy 8)

Figure 9.1a - Change in Welfare by Generation Debt .10 (Tax) (Policy 9)
Linear Production Function

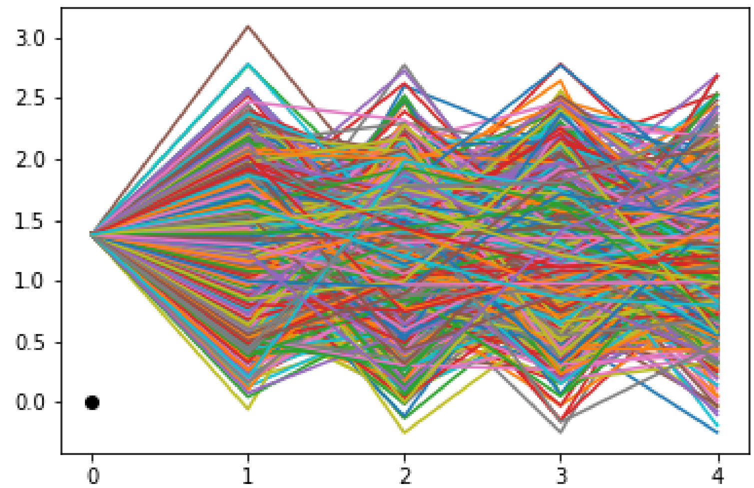


Figure 9.2a - Debt (Share Savings) Debt .10 (Tax) (Policy 9)
Linear Production Function

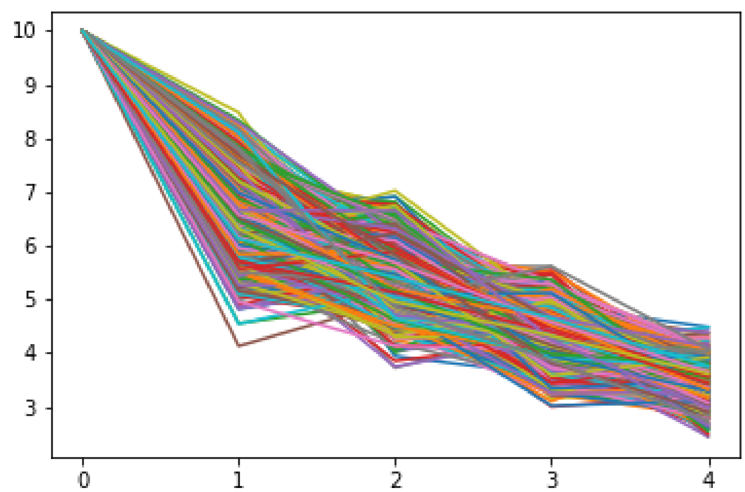


Figure 9.1b - Change in Welfare by Generation Debt .10 (Tax) (Policy 9)
Cobb-Douglas Production Function

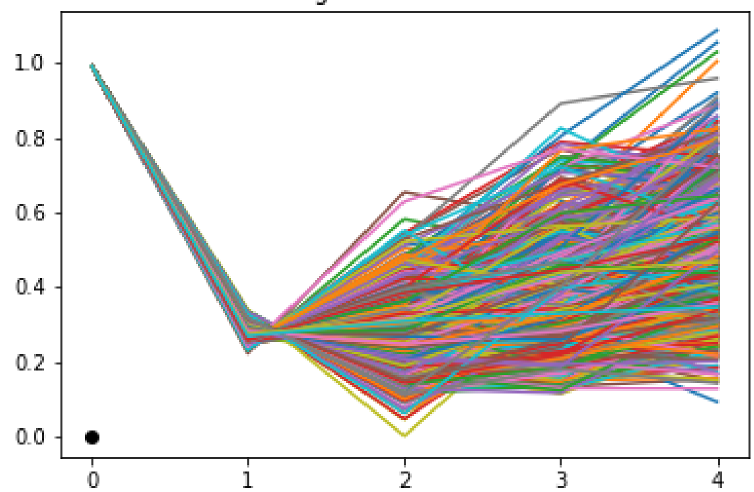


Figure 9.2b - Debt (Share Savings) Debt .10 (Tax) (Policy 9)
Cobb-Douglas Production Function

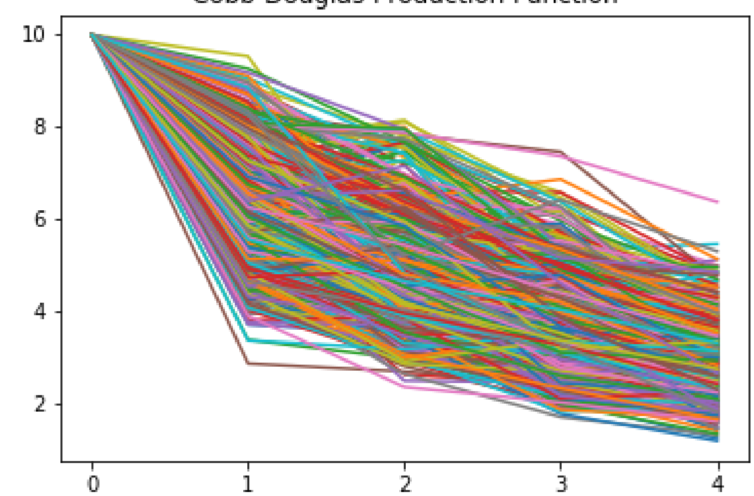


Figure 9: Short-term implications of debt rollovers and subsidies (Policy 9)

10.1a - Change in Welfare by Generation Extended Debt .10 (Tax) (Poli
Linear Production Function

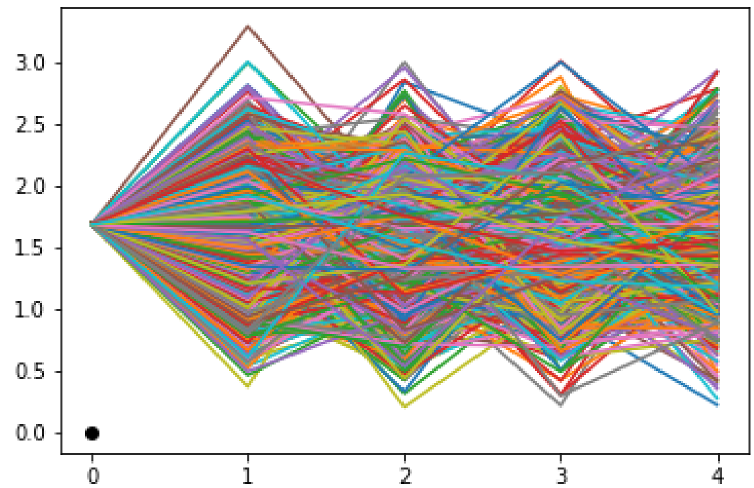
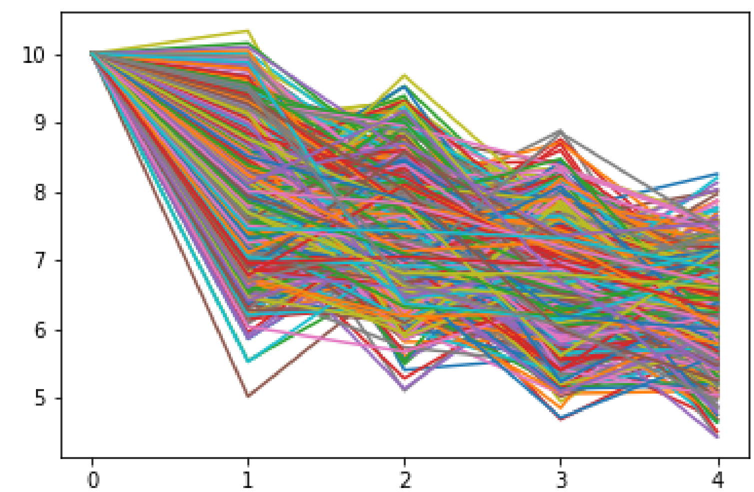
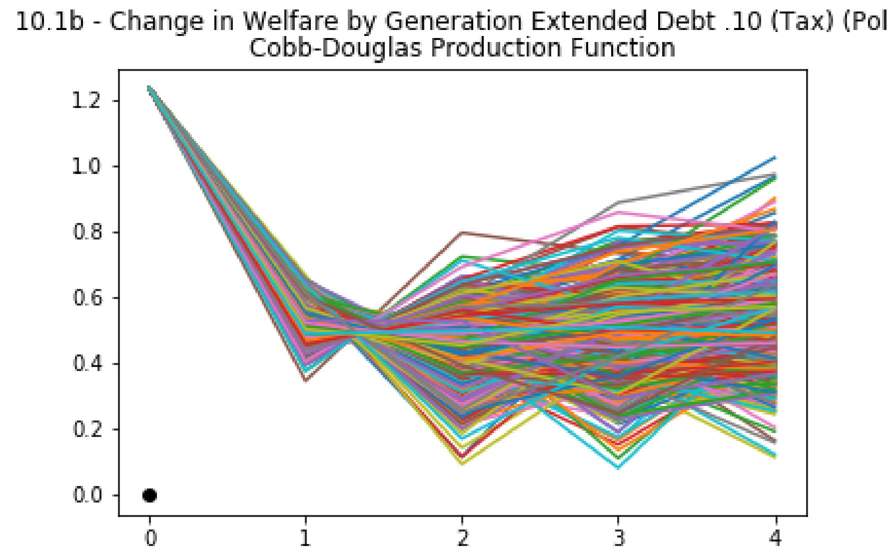


Figure 10.2a - Debt (Share Savings) Extended Debt .10 (Tax) (Policy 10)
Linear Production Function





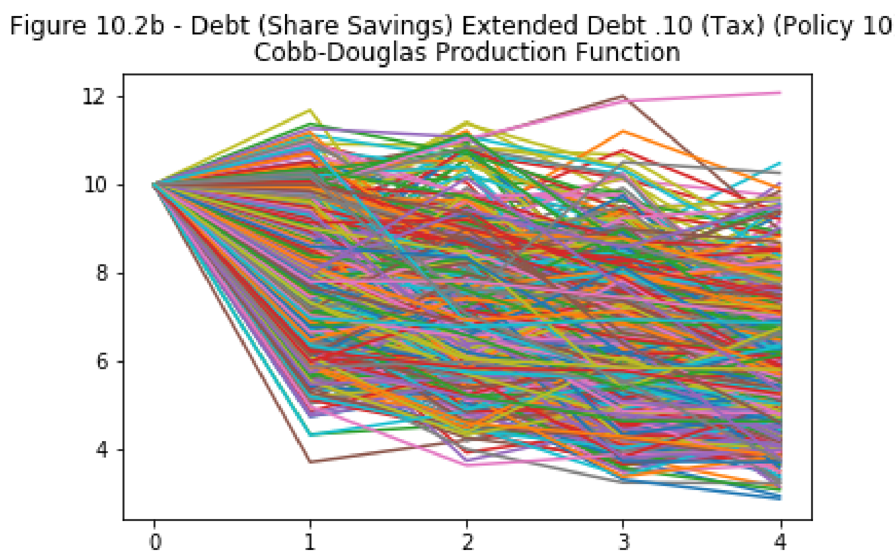


Figure 10: Short-term implications of extended debt rollovers and subsidies (Policy 10)

Appendix - Derivations

A) Description of the Model

Assume the representative agent maximizes an Epstein-Zin utility function:

$$\mathbb{U}_t = (1-\beta)u\left(C_t^y\right) + \frac{\beta}{1-\gamma}u(\mathbb{E}_t\left[(C_{t+1}^o)^{1-\gamma}\right])$$

Where: $u(C) = \log(C)$

With respect to:

$$\begin{aligned} C_t^y + I_t + D_t &= W_t + X - T_t \\ K_t &= I_t + (1-\delta)K_{t-1} \\ C_{t+1}^o &= R_{t+1}K_t + R_t^f D_t + T_{t+1} \\ Y_t &= A_t F(K_{t-1}, L_t) \\ \log(A) &\sim \mathcal{N}(\mu, \sigma) \end{aligned}$$

Where C_t^y and C_{t+1}^o respectively denote consumption when young and old, I_t is investment in physical capital, D_t is investment in the safe asset, W_t is the wage, X is an initial non-stochastic endowment, T_t and T_{t+1} denote intergenerational transfers (transfers can be stochastic or not), A_t is a log-normally distributed productivity shock, and R_{t+1} and R_t^f denote respectively the return to physical capital and risk-free asset. Assume that there is full depreciation after one period ($\delta = 1$) so that $K_t = I_t$.

Factors earn their marginal return:

$$\begin{aligned} W_t &= A_t F_L(K_{t-1}, L_t) \\ R_t &= A_t F_K(K_{t-1}, L_t) \end{aligned}$$

In the baseline scenario, there is no government intervention: $T_t = D_t = 0\ \forall t$. The paper discusses the welfare implications of two types of policy intervention. The government can start a social security system and sets the level of transfers T_t accordingly. Alternatively, the government can start to issue and rollover public debt D_t. Absent default, the debt dynamics equation is $D_{t+1} = R_t^f D_t$.[6]

[6] If the debt rollover fails, the government taxes the *young* so as to bring the debt back to its target value D^*. Formally, the *young* budget constraint is $C_t^y + I_t + D_t = W_t + X - T_t - \theta_t$ where $\theta_t = 0$ if $D_t < \bar{D}$ and $\theta_t = D_t - D^*$ if $D_t \geq \bar{D}$. For simplicity I assume $\theta_t = 0\ \forall t$ in the appendix. This does not change the main results but simplifies the exposition.

B) General solution with debt and transfers: $D_t \geq 0$; $T_t \neq 0$.

If the government runs a social security system and/or issues debt, the maximization problem can be rewritten:

$$\max_{I_t,D_t} \mathbb{U} = (1-\beta)\ \log(W_t+X-T_t-I_t-D_t)+\frac{\beta}{1-\gamma}\ \log(\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{1-\gamma}]) \tag{1}$$

The first order condition with respect to I_t is:

$$\frac{1-\beta}{(W_t+X-T_t-I_t-D_t)} = \frac{\beta}{1-\gamma}\frac{(1-\gamma)\mathbb{E}_t[R_{t+1}(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{1-\gamma}]} \tag{2}$$

Similarly, the first order condition with respect to D_t is:

$$\frac{1-\beta}{(W_t+X-T_t-I_t-D_t)} = \frac{\beta}{1-\gamma}\frac{(1-\gamma)\mathbb{E}_t[R_t^f(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{1-\gamma}]} \tag{3}$$

As R_t^f is known at time t it can be taken out of the expectation term:

$$\frac{1-\beta}{(W_t+X-T_t-I_t-D_t)} = \beta\frac{R_t^f\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{1-\gamma}]} \tag{4}$$

The left hand side of equations (2) and (4) are equal, thus:

$$\beta\frac{R_t^f\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{1-\gamma}]} = \beta\frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{1-\gamma}]} \tag{5}$$

Diving by $\beta\frac{\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{1-\gamma}]}$ on both sides:

$$R_t^f = \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t+R_t^f D_t+T_{t+1})^{-\gamma}]} \tag{6}$$

The environment is a closed economy. Market clearing requires that young households hold all the debt inelastically supplied by the government. Put differently, young households are constrained in their quantity of safe asset holdings, but their demand function is used to determine the equilibrium rate of return they require on this asset given the quantity D_t. If there is debt in the economy ($D_t > 0$), then the investment in physical capital I_t and the risk-free rate R_t^f consistent with a given debt level D_t are obtained by solving simultaneously equations (2) and (6) derived from the first order conditions.

C) Solution with transfers but no debt: $D_t = 0$; $T_t \neq 0$.

If there is no debt in the economy ($D_t = 0\ \forall t$), then equation (2), which is used to solve for I_t, simplifies to:

$$\frac{1-\beta}{(W_t + X - T_t - I_t)} = \beta \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t + T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t + T_{t+1})^{1-\gamma}]} \tag{7}$$

If there is no debt, the risk-free rate is not used to derive the optimal investment in physical capital. Yet, the shadow risk-free rate consistent with young agents being indifferent between investing at the margin or not in risk-free debt is given by (6), which simplifies to:

$$R_t^f = \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t + T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t + T_{t+1})^{-\gamma}]} \tag{8}$$

D) Calibration without government: $D_t = 0$; $T_t = 0$.

The calibration follows Blanchard (2019). Assume for simplicity that $X = 0$. Again, this does not change the main results but simplifies the exposition. If there is no debt and no transfers in the economy ($D_t = T_t = 0\ \forall t$), then equation (2), which is used to solve for I_t, simplifies to:

$$\frac{1-\beta}{(W_t - I_t)} = \beta \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t)^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t)^{1-\gamma}]} \tag{9}$$

After some algebra this leads to the following optimal investment decision:

$$I_t = \beta W_t \tag{10}$$

Similarly, given the separable log utility specification, the optimal consumption decision is:

$$C_t^y = (1-\beta) W_t \tag{11}$$

Absent government intervention, the risk-free rate is not used to derive the optimal investment in physical capital. Yet, the shadow risk-free rate consistent with young agents being indifferent between investing at the margin or not in risk-free debt is given by (6), which simplifies to:

$$R_t^f = \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t)^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t)^{-\gamma}]} \tag{12}$$

To make further analytic progress, we need to specify a production function.

i) Linear production function:

$$Y_t = A_t(\alpha K_{t-1} + (1-\alpha)L_t) \tag{13}$$

Factors earn their marginal product in equilibrium:

$$R_t = \alpha A_t \tag{14}$$

$$W_t = (1-\alpha)A_t \tag{15}$$

Equation (12) can be rewritten:

$$R_t^f = \frac{\mathbb{E}_t[(\alpha A_{t+1})^{1-\gamma}]}{\mathbb{E}_t[(\alpha A_{t+1})^{-\gamma}]} \tag{16}$$

Taking α out of the expectation operator, and using the fact that if X is log-normally distributed then $E[X^n] = e^{n\mu + \frac{1}{2}n^2\sigma^2}$, we obtain:

$$R_t^f = \alpha e^{\mu + \frac{1}{2}\sigma^2 - \gamma\sigma^2} \tag{17}$$

From equation (14), we obtain:

$$\mathbb{E}[R_{t+1}] = \alpha e^{\mu + \frac{1}{2}\sigma^2} \tag{18}$$

From equation (17) and (18), we obtain the log equity premium:

$$\ln(\mathbb{E}[R_{t+1}]) - \ln(R_t^f) = \gamma\sigma^2 \tag{19}$$

We obtain the steady state value of capital from equations (10) and (15):

$$\mathbb{E}[K_t] = \bar{K} = \beta(1-\alpha)e^{\mu+\sigma^2/2} \tag{20}$$

$\mathbb{E}[R_{t+1}]$ does not depend on K_t and thus does not depend on β, but can be calibrated with μ while the equity premium, and thus $\mathbb{E}[R_t^f]$, depends on γ.

ii) Cobb-Douglas production function:

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{21}$$

Factors earn their marginal product in equilibrium:

$$R_t = A_t \alpha K_{t-1}^{\alpha-1} \tag{22}$$

$$W_t = A_t (1-\alpha) K_{t-1}^{\alpha} \tag{23}$$

Equation (12) can be rewritten:

$$R_t^f = \frac{\mathbb{E}_t[(\alpha K_t^{\alpha-1} A_{t+1})^{1-\gamma}]}{\mathbb{E}_t[(\alpha K_t^{\alpha-1} A_{t+1})^{-\gamma}]} \tag{24}$$

Taking $\alpha K_t^{\alpha-1}$ out of the expectation operator, and using the fact that if X is log-normally distributed then $E[X^n] = e^{n\mu + \frac{1}{2} n^2 \sigma^2}$, we obtain:

$$R_t^f = \alpha K_t^{\alpha-1} e^{\mu + \frac{1}{2}\sigma^2 - \gamma\sigma^2} \tag{25}$$

We derive the steady state value of capital from equations (10) and (23):

$$K_t = \beta A_t (1-\alpha) K_{t-1}^{\alpha} \tag{26}$$

By taking log on both sides:

$$k_{t+1} = \log[\beta(1-\alpha)] + \alpha k_t + \log(A_t) \tag{27}$$

Evaluating at $k_{t+1} = k_t$, the expectation and variance are:

$$\mathbb{E}[k_t] = \frac{\log[\beta(1-\alpha)] + \mu}{1-\alpha} \tag{28}$$

$$\mathbb{V}[k_t] = \frac{\sigma^2}{1-\alpha^2} \tag{29}$$

Thus, the steady state value of capital is:

$$\mathbb{E}[K_t] = \bar{K} = e^{\mathbb{E}k} e^{\frac{\mathbb{V}k}{2}} = e^{\left(\frac{\log[\beta(1-\alpha)]+\mu}{1-\alpha} + \frac{\sigma^2/2}{1-\alpha^2}\right)} \tag{30}$$

Taking log of equation (25):

$$r_t^f = \log(\alpha) + (\alpha - 1)k_t + \mu + (\frac{1}{2} - \gamma)\sigma^2 \tag{31}$$

Using (28), the expectation and variance are:

$$\mathbb{E}[r_t^f] = \log \frac{\alpha}{\beta(1-\alpha)} + (\frac{1}{2} - \gamma)\sigma^2 \tag{32}$$

$$\mathbb{V}[r_t^f] = \frac{1-\alpha}{1+\alpha}\sigma^2 \tag{33}$$

The unconditional expected value of the risk-free rate is:

$$\mathbb{E}[R_t^f] = \frac{\alpha}{\beta(1-\alpha)} e^{(\frac{\sigma^2}{1+\alpha} - \gamma\sigma^2)} \tag{34}$$

Similarly, the unconditional expected value of the risky rate is:

$$\mathbb{E}[R_{t+1}] = \frac{\alpha}{\beta(1-\alpha)} e^{(\frac{\sigma^2}{1+\alpha})} \tag{35}$$

From equation (34) and (35), we obtain the log equity premium:

$$\ln(\mathbb{E}[R_{t+1}]) - \ln(R_t^f) = \gamma\sigma^2 \tag{36}$$

$\mathbb{E}[R_{t+1}]$ does not depend on μ, but can be calibrated with β while the equity premium, and thus $\mathbb{E}[R_t^f]$, depends on γ.

E) Methodology

This paper evaluates the welfare implications of debt and transfers for different combinations of $\mathbb{E}[R_{t+1}]$ and $\mathbb{E}[R_t^f]$ absent government intervention. First, for every combination of both average rates, I find the corresponding parameters μ and γ from equations (17) and (18) if the production is linear, and the corresponding parameters β and γ from equations (34) and (35) if the production is Cobb-Douglas. Then, I use those parameters to simulate the economy for multiple periods, with and without policy intervention. The optimal investment decision and the shadow risk-free rate are computed numerically every period by solving simultaneously equations (2) and (6), which hold for any specification. Finally, I compare the welfare outcomes for every combination of parameters.