Welfare Implications of Debt and Transfers in a Low Safe Rate Environment

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Outline

I. Introduction

II. The Stochastic OLG Model

III. Long-Run Welfare

IV. Short-term Welfare

V. Conclusion

A Low Safe Rate Environment

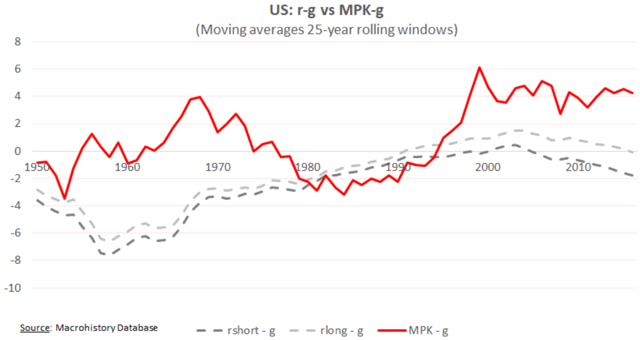

Figure: Nominal interest rates, GDP growth, and stock returns

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- Is the economy *dynamically inefficient* in such environment?
  Can a social planner generate a Pareto welfare improvement?
  Yes.
- Does *dynamic inefficiency* imply an over-accumulation of capital?
  No.

Literature

- On low rates: Mehrotra (2018), Rachel and Summers (2019)
- On *dynamic inefficiency* in OLG models:
  1. Samuelson (1958), Diamond (1965): $r^f < g$
  2. Abel et al. (1989): $r_t^K K_t \leq I_t \; \forall t$
- On transfers and debt policies: Ball et al. (1998), Blanchard (2019), DeLong and Waldmann (2019)
- On risk-allocation in OLG models: Bohn (1998), Shiller (1999)

OLG model under uncertainty: Households

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$$\mathbb{U}_t = (1-\beta)\ln C_t^y + \beta \frac{1}{1-\gamma} \ln \mathbb{E}[(C_{t+1}^o)^{1-\gamma}] \tag{1}$$

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- **Budget constraints**. The budget constraints are:

$$C_t^y + I_t + D_t = W_t + X - T_t - \theta_t \quad (2)$$

$$C_{t+1}^o = R_{t+1} I_t + R_t^f D_t + T_{t+1} \quad (3)$$

OLG model under uncertainty: Firms

- **Production**. Two cases. In the Cobb-Douglas case:

$$Y_t = A_t K_{t-1}^{\alpha} L^{1-\alpha} \tag{4}$$

In the linear case:

$$Y_t = A_t(\alpha K_{t-1} + (1-\alpha)L) \tag{5}$$

where $L = 1$, and A_t iid s.t. $\log(A) \sim \mathcal{N}(\mu, \sigma)$.

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- **Capital motion**. Capital fully depreciates after one period: $\delta = 1$.

$$K_t = I_t \tag{6}$$

OLG model under uncertainty: Government

- **Policies**. The government can:
  1. Implement inter-generational transfers and set T_t every period.
  2. Issue risk-free debt D_0 and rollover debt D_t at the real interest rate R_t^f every period. Absent default:

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- **Default**. The tax is: $\theta_t = D_t - D^*$ if $D_t > \bar{D}$ and $\theta_t = 0$ otherwise. This strong assumption ensures that debt is perceived as perfectly safe by households.

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The first order condition with respect to I_t is:

$$\frac{1-\beta}{(W_t + X - T_t - I_t - D_t - \theta_t)} = \beta \frac{\mathbb{E}_t[R_{t+1}(R_{t+1} I_t + R_t^f D_t + T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1} I_t + R_t^f D_t + T_{t+1})^{1-\gamma}]} \quad (9)$$

Similarly, the first order condition with respect to D_t is:

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The safe interest rate, which is determined by the equilibrium of inelastic supply from the government and the demand from the *young* is:

$$R_t^f = \frac{\mathbb{E}_t[R_{t+1}(R_{t+1}I_t + R_t^f D_t + T_{t+1})^{-\gamma}]}{\mathbb{E}_t[(R_{t+1}I_t + R_t^f D_t + T_{t+1})^{-\gamma}]} \quad (11)$$

Calibration: No Government Intervention

Closed-form solutions for steady-state rates:

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Closed-form solutions for steady-state rates:

- In the linear case:
  $R_t^f = \alpha e^{\mu + \frac{1}{2}\sigma^2 - \gamma\sigma^2}$ and $\mathbb{E}[R_{t+1}] = \alpha e^{\mu + \frac{1}{2}\sigma^2}$
- In the Cobb-Douglas case:
  $R_t^f = \frac{\alpha}{\beta(1-\alpha)} e^{\left(\frac{\sigma^2}{1+\alpha} - \gamma\sigma^2\right)}$ and $\mathbb{E}[R_{t+1}] = \frac{\alpha}{\beta(1-\alpha)} e^{\left(\frac{\sigma^2}{1+\alpha}\right)}$
- In both cases, the log equity premium is:
  $\ln(\mathbb{E}[R_{t+1}]) - \ln(R_t^f) = \gamma\sigma^2$

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- Calibration: $\alpha = 1/3$; $\sigma = 0.2$; $X = W^*$;
  $\beta = 0.325$ in linear; $\mu = 3$ in Cobb-Douglas (Blanchard (2019))

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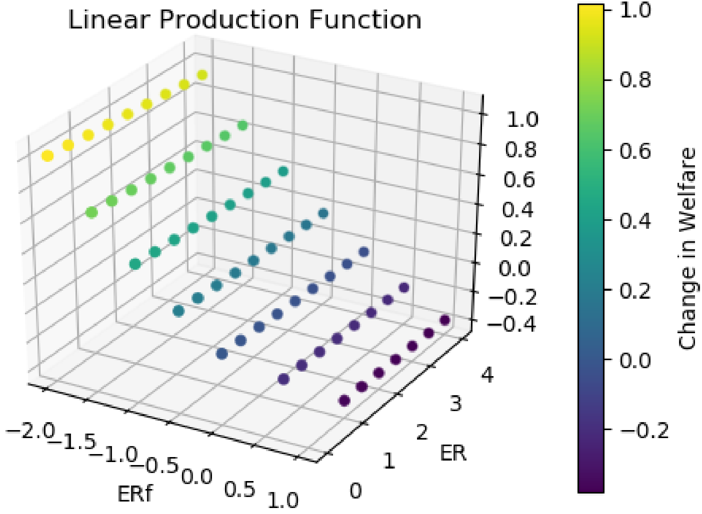
  $\beta = 0.325$ in linear; $\mu = 3$ in Cobb-Douglas (Blanchard (2019))
- Specify pairs of values for μ (resp. β) and γ in the linear (resp. Cobb-Douglas) case consistent with $\mathbb{E}[R] \in [0\%; 4\%]$ and $R^f \in [-2\%; 1\%]$

PAYGO: Fixed transfers, Blanchard (2019)

- **Policy 1**: $T_t = \tau I^* \; \forall t$. Calibration: $\tau = 5\%$.

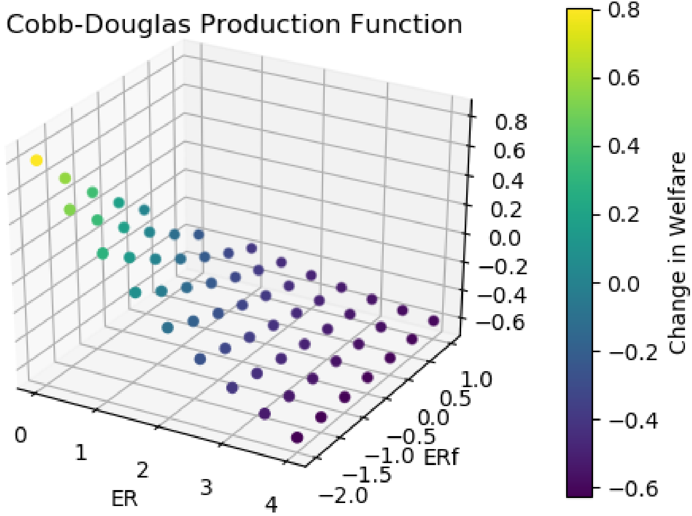
PAYGO: Fixed transfers, Blanchard (2019)

Figure 1a - Fixed transfer equal to 5% ISS (Policy 1)



PAYGO: Fixed transfers, Blanchard (2019)

Figure 1b - Fixed transfer equal to 5% ISS (Policy 1)



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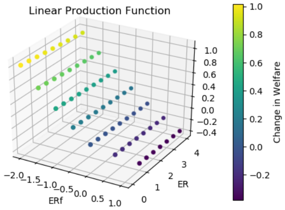

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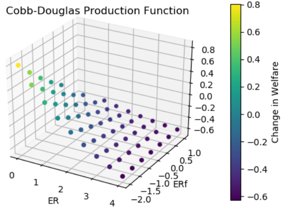


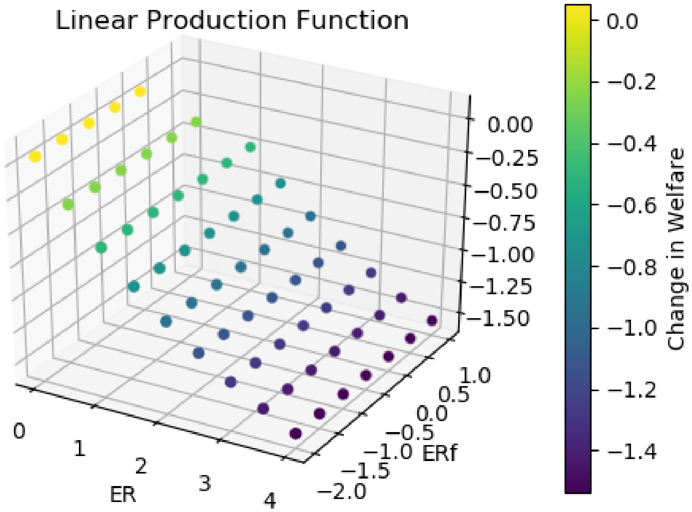
- Intuition: the policy offers a safe asset with a net return of 0% while agents were indifferent at the margin between investing in the risky or the risk-free asset
- In GE, lower capital accumulation, higher $\mathbb{E}[R]$ and thus higher R^f, policy less attractive

PAYGO: Stochastic transfers

- **Policy 3**: $T_t = \tau W_t \ \forall t$. Calibration: $\tau = 5\%$.

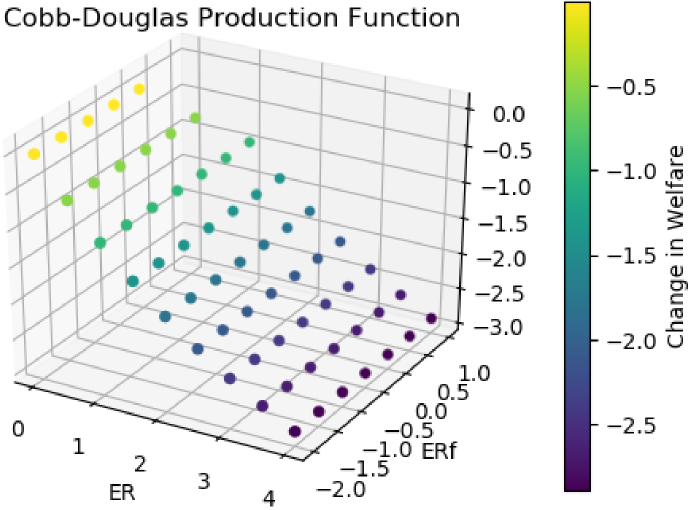
PAYGO: Stochastic transfers

Figure 3a - Stochastic transfer equal to 5% W (Policy 3)



PAYGO: Stochastic transfers

Figure 3b - Stochastic transfer equal to 5% W (Policy 3)



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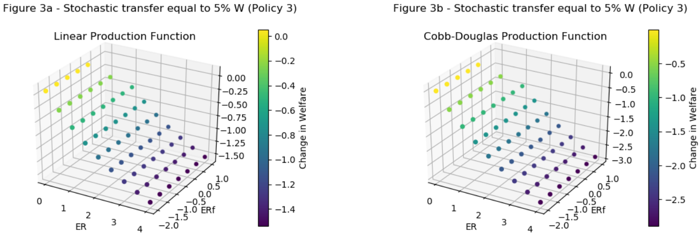
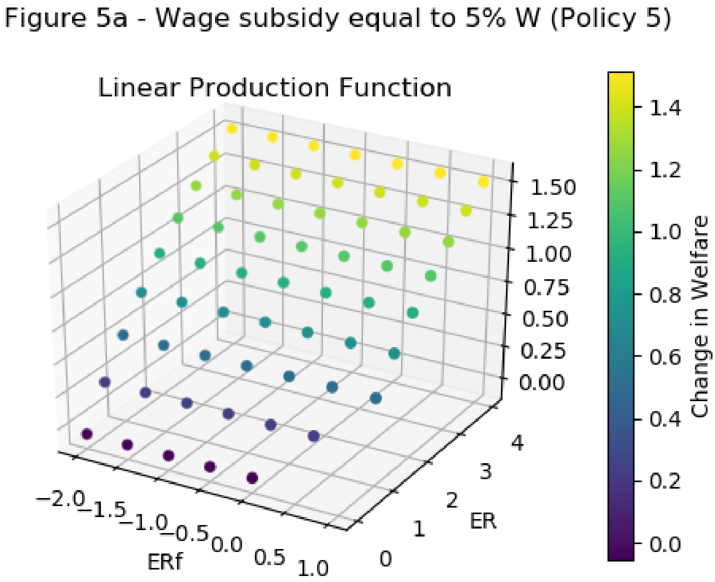


- Intuition: the policy offers a risky asset with an average expected net return of 0% and same uncertainty (returns to capital and labor perfectly correlated)
- In GE, lower capital accumulation, higher $\mathbb{E}[R]$, policy less attractive

Wage Subsidies

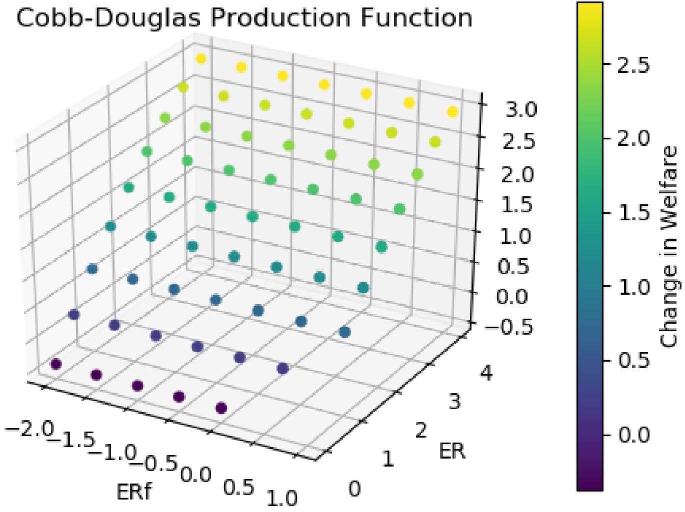
- **Policy 5**: $T_t = \tau W_t \; \forall t$. Calibration: $\tau = -5\%$.

Wage Subsidies



Wage Subsidies

Figure 5b - Wage subsidy equal to 5% W (Policy 5)



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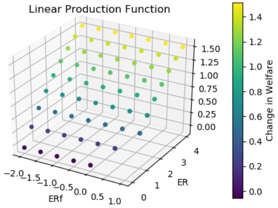
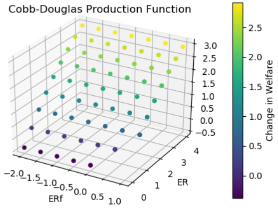


Figure 5b - Wage subsidy equal to 5% W (Policy 5)



- Intuition: the policy offers the possibility to invest some extra income at an average expected return $\mathbb{E}[R]$ while the average net expected cost of this extra income is 0%
- Also, offers income diversification!
- In GE, higher capital accumulation, policy more attractive if $\mathbb{E}[R]$ high enough

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- If the policy intervention takes the form of a PAYGO system with stochastic transfers then only the average risky rate matters to assess steady-state welfare implications
- Tension between policies that improve welfare of future generations at the expense of current generations, and *vice versa*

Calibration

- Simulate 1,000 paths of the economy with and without intervention.
- Study the welfare implications for up to 5 generations (125 years)
- Calibration: $\mathbb{E}\,R = 2\%$ and $R^f = -1\%$ as in Blanchard (2019); $\bar{D} = 0.1725 I^*$ and $D^* = 0.4 D_0$

Debt Rollovers, Blanchard (2019)

- **Policy 7**: $T_t = 0 \ \forall t$; $D_0 = \kappa I^*$. Calibration: $\kappa = 15\%$.

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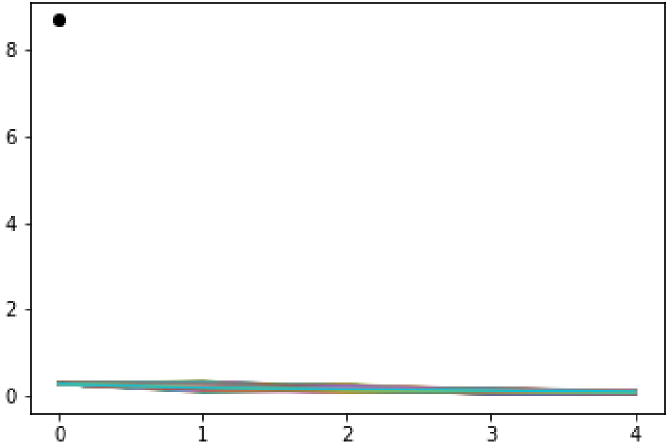
Figure 7.1a - Change in Welfare by Generation Debt .15 (Policy 7)
Linear Production Function



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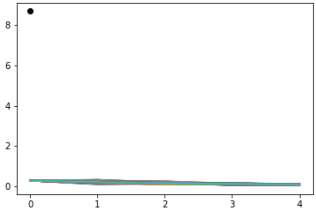
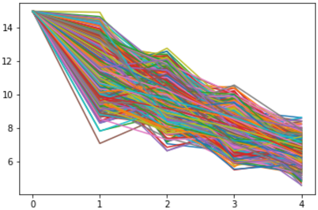


Figure 7.2a - Debt (Share Savings) Debt .15 (Policy 7)
Linear Production Function



- For current *old* clear positive and large effect (black dot), slightly larger utility for later generations (less risky portfolio)
- Debt rollovers typically do not fail and welfare is increased throughout

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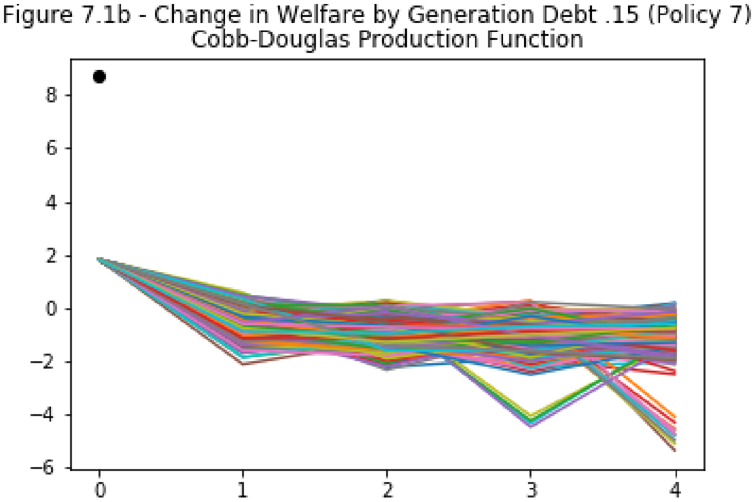

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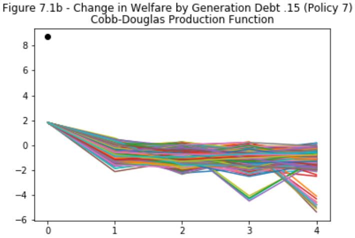
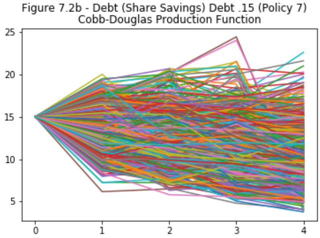


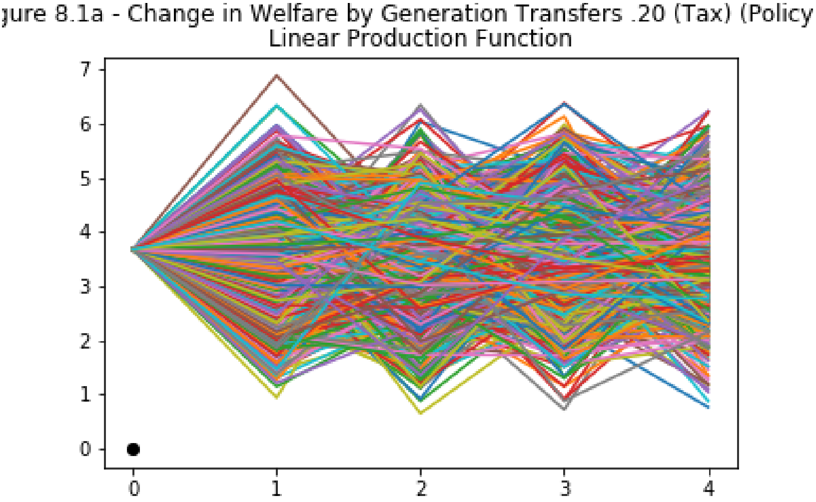


- Welfare still goes up for the first young generation (by about 2 percent), but is typically negative thereafter
- In the case of unsuccessful rollovers, the adjustment implies a larger welfare loss when it happens

Transfers and Subsidies

- **Policy 8**: $T_t = \tau_I I^* + \tau_W W_t \ \forall t$. With $\tau_I = 20\%$ and $\tau_W = -\frac{\tau_I I^*}{W^*}\%$

Transfers and Subsidies



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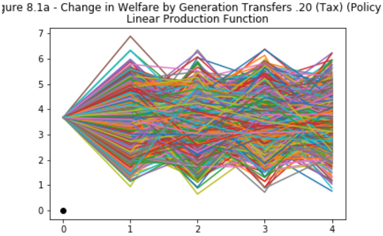

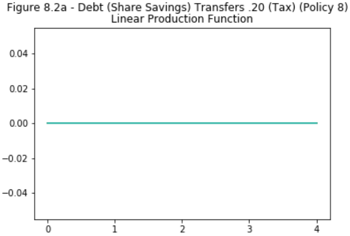

- In all simulations, welfare goes up for all generations (by about 3 percent on average) except current *old* who are indifferent: Pareto improvement
- One-time drop in capital accumulation

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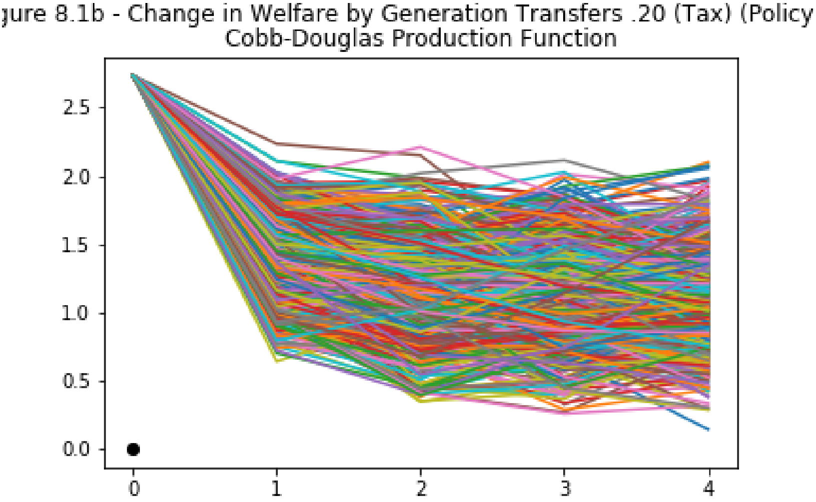

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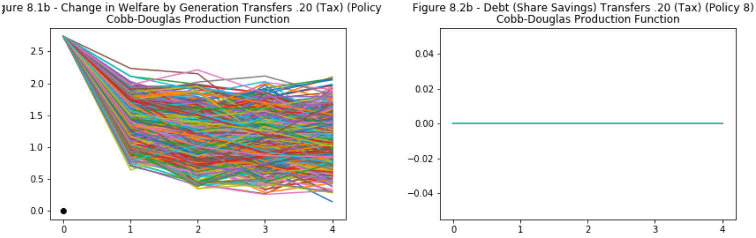


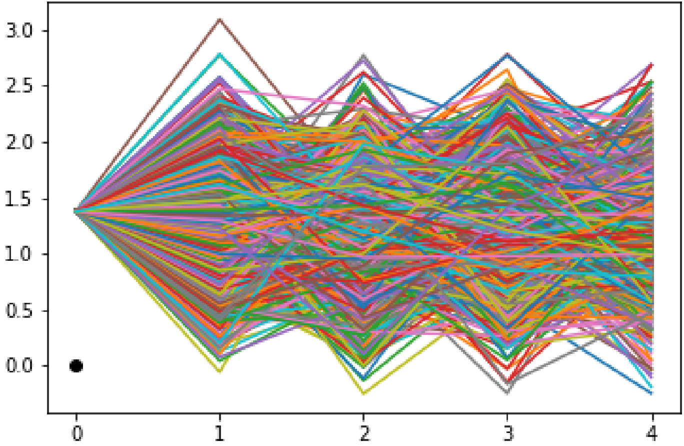
- Again, a Pareto improvement
- Transfers imply lower capital accumulation, but the wage subsidy fosters investment (see proof) thus limiting the adverse price effect

Debt Rollovers and Subsidies

- **Policy 9**: $T_t = \tau W_t \; \forall t$; $D_0 = \kappa I^*$. Calibration: $\kappa = 10\%$; $\tau = -\frac{D_0}{W^*}$

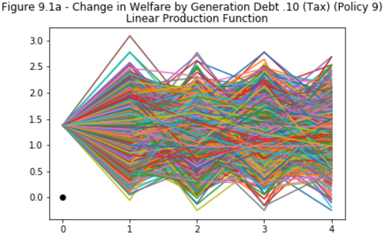
Debt Rollovers and Subsidies

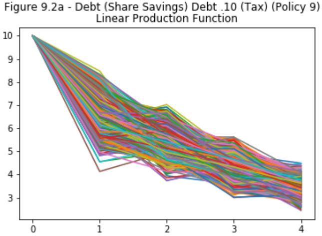
Figure 9.1a - Change in Welfare by Generation Debt .10 (Tax) (Policy 9)
Linear Production Function



Debt Rollovers and Subsidies

- **Policy 9**: $T_t = \tau W_t \ \forall t$; $D_0 = \kappa I^*$. Calibration: $\kappa = 10\%$; $\tau = -\frac{D_0}{W^*}$



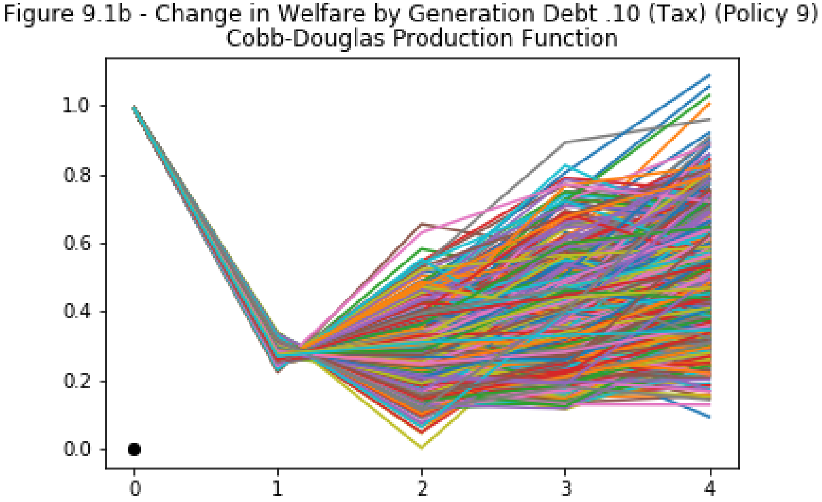


- Two effects: $R^f < 1$, but debt level and crowding out effect vanish over time (here not operative)
- In a few simulations some generations experience a decrease in welfare, but the decrease is small

Debt Rollovers and Subsidies

- **Policy 9**: $T_t = \tau W_t \ \forall t$; $D_0 = \kappa I^*$. Calibration: $\kappa = 10\%$; $\tau = -\frac{D_0}{W^*}$

Debt Rollovers and Subsidies



Debt Rollovers and Subsidies

- **Policy 9**: $T_t = \tau W_t \ \forall t$; $D_0 = \kappa I^*$. Calibration: $\kappa = 10\%$; $\tau = -\frac{D_0}{W^*}$

Figure 9.1b - Change in Welfare by Generation Debt .10 (Tax) (Policy 9)
Cobb-Douglas Production Function

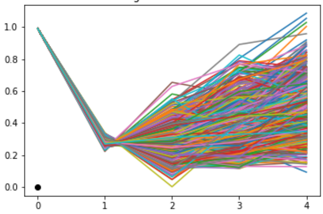
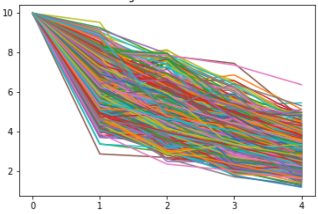


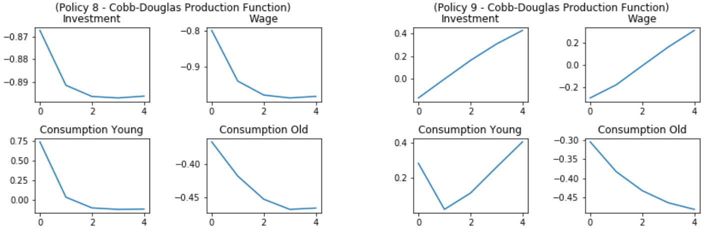
Figure 9.2b - Debt (Share Savings) Debt .10 (Tax) (Policy 9)
Cobb-Douglas Production Function



- Two effects: $R^f < 1$, but debt level and crowding out effect vanishes over time

Debt Rollovers and Subsidies

Comparing the effects on Wages and Investment

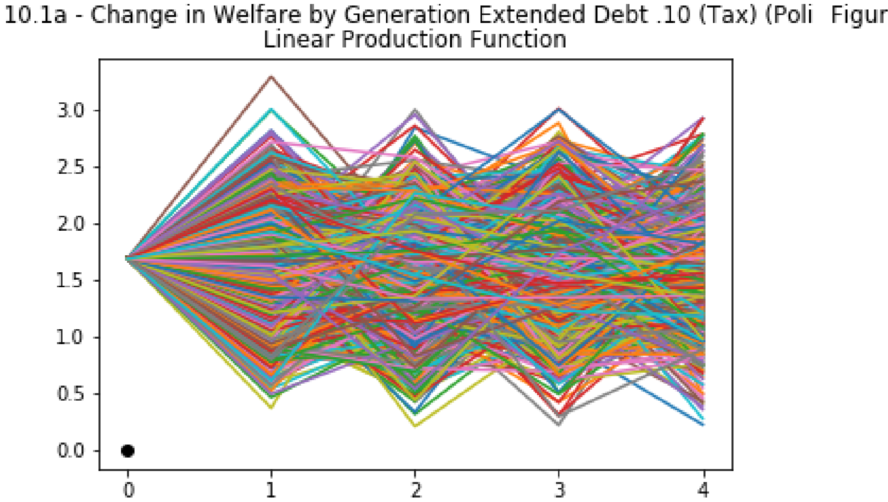


- Transfers + Subsidy: lower SS values
- Debt Rollover + Subsidy: higher SS values

Extended Debt Rollovers and Subsidies

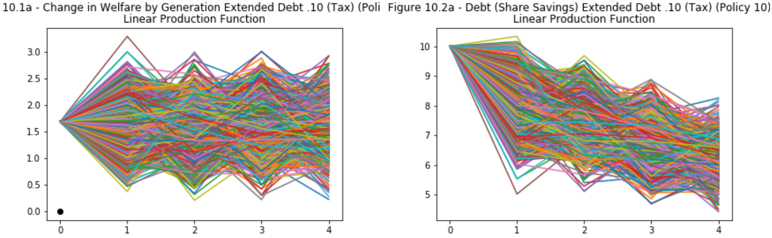
- **Policy 10**: $T_t = \tau W_t \; \forall t;\; D_0 = \kappa I^*;\; D_{t+1} = R_t^f D_t + \varkappa D_0$.
  Calibration: $\kappa = 10\%;\; \varkappa = 7.5\%;\; \tau = -\frac{D_0}{W^*}$

Extended Debt Rollovers and Subsidies



Extended Debt Rollovers and Subsidies

- **Policy 10**: $T_t = \tau W_t \ \forall t$; $D_0 = \kappa I^*$; $D_{t+1} = R_t^f D_t + \varkappa D_0$.
  Calibration: $\kappa = 10\%$; $\varkappa = 7.5\%$; $\tau = -\frac{D_0}{W^*}$

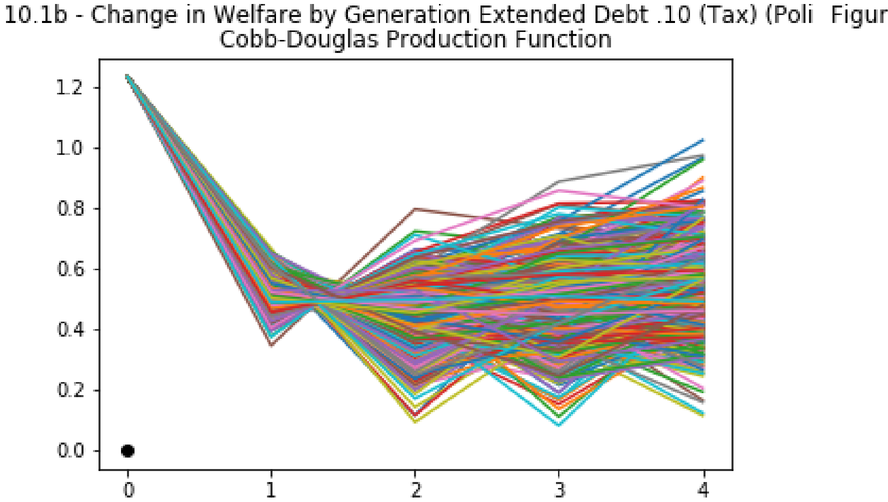


- Now it is a Pareto improvement, all generations benefit in all simulations
- Debt more persistent than simple debt rollover (more akin to fixed transfers), but still decreases

Extended Debt Rollovers and Subsidies

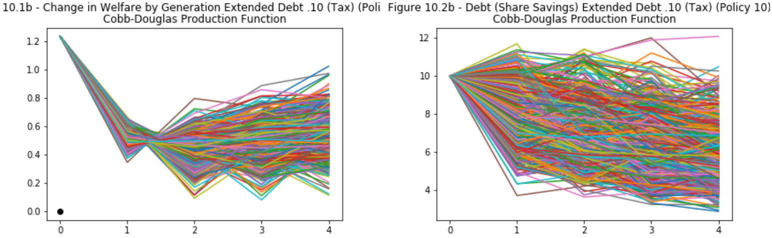
- **Policy**: $T_t = \tau W_t \ \forall t$; $D_0 = \kappa I^*$; $D_{t+1} = R_t^f D_t + \varkappa D_0$.
  Calibration: $\kappa = 10\%$; $\varkappa = 7.5\%$; $\tau = -\frac{D_0}{W^*}$

Extended Debt Rollovers and Subsidies



Extended Debt Rollovers and Subsidies

- **Policy**: $T_t = \tau W_t \; \forall t$; $D_0 = \kappa I^*$; $D_{t+1} = R_t^f D_t + \varkappa D_0$.
  Calibration: $\kappa = 10\%$; $\varkappa = 7.5\%$; $\tau = -\frac{D_0}{W^*}$



- Again it is a Pareto improvement
- Debt more persistent than simple debt rollover (more akin to fixed transfers), but still decreases, thus higher SS capital level

Conclusion

- If PAYGO system with stochastic transfers then only risky rate matters
- The economy is likely to be *dynamically inefficient* in such low rate environment: PAYGO system with fixed transfers and wage subsidies are Pareto welfare improving
- The combination of a debt rollover and a wage subsidy: Pareto welfare improvement and higher level of steady-state capital
- Would be interesting to understand how the optimal size in the initial debt increase varies with different combinations of the average safe and risky rates, or to include business cycle considerations

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Wage Subsidies - Proof

With linear production function:

Optimal investment decision:

$$I_t^L = \beta W_t(1-\tau) - \frac{1-\alpha}{\alpha}\tau(1-\beta)$$

Result 1. Assuming $0 < \alpha, \beta < 1$, $I_t^L > I_t$ if and only if $\tau < 0$. Conversely, $I_t^L < I_t$ if and only if $\tau > 0$.

Consumption when *young* and *old* absent government intervention are:

$$C_t^y = (1-\beta)W_t$$

$$C_{t+1}^o = R_{t+1} I_t$$

Consumption when *young* and *old* after the government intervention are:

$$C_t^{y,L} = W_t(1-\tau) - I_t^L = (1-\beta)[W_t(1-\tau) + \frac{1-\alpha}{\alpha}\tau]$$

$$C_{t+1}^{o,L} = R_{t+1} I_t^L + \tau W_{t+1}$$

Result 2. Consider two cases:

- If $R_t \geq 1$ then $C_t^{y,L} \geq C_t^y$ and $C_{t+1}^{o,L} \geq C_{t+1}^o$ if and only if $\tau < 0$.
- If $R_t \leq 1$ then $C_t^{y,L} \geq C_t^y$ and $C_{t+1}^{o,L} \geq C_{t+1}^o$ if and only if $\tau > 0$.