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18-7 Growth-indexed Bonds and Debt Distribution: Theoretical Benefits and Practical Limits

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Abstract

Sovereign state-contingent bonds, in particular growth-indexed bonds (GIBs), have rarely been issued in practice despite their theoretical benefits. This paper provides support for this apparent sovereign noncontingency puzzle by deriving the impact of GIBs on the upper tail of the distribution of the public debt-to-GDP ratio. Although this impact varies importantly across countries and indexation formulas, empirical estimates show there is almost no reduction in the upper tail of the distribution under the realistic assumption that GIBs only represent 20 percent of the stock of debt. Moreover, a sustained premium of 100 basis points would actually increase the upper tail of the distribution for most countries.

JEL Codes: E62, F34, F45, H63

Keywords: Growth-indexed bonds, State-contingent bonds, Debt sustainability, Monte Carlo simulations

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1 Introduction

Advanced economies experienced a massive increase in their public debt-to-GDP ratios after the last global financial crisis. The deep recession and slow economic recovery raised concerns about debt sustainability in some European countries, creating a vicious cycle: Negative shocks on output and on public finances reduced debt sustainability, which translated into higher borrowing costs, thus worsening debt sustainability in those countries even more. To contain the impact of increasing borrowing costs, governments went through fiscal consolidation, which acted as a drag on economic activity. In Spain, for example, the debt-to-GDP ratio dramatically increased[1] from 35 percent in 2007 to almost 100 percent in 2014, while it increased from 65 percent to 95 percent in France over the same period (figure 1).

Public debt-to-GDP ratios are expected to remain at historically high levels in most advanced economies. Although such levels seem sustainable, negative shocks to GDP growth and/or interest rates may put debt ratios on an unsustainable path. Against this background, researchers and policymakers have advanced different proposals to break the vicious cycle described above. An old idea introduced in its current version by Shiller (1993), the issuance by sovereigns of long-term debt instruments with a return indexed to output, has recently regained momentum in policy circles.[2] While sovereigns traditionally issue debt at fixed interest rates, growth-indexed bonds (GIBs) pay a fixed principal and a time-varying interest rate determined by a specific indexation formula. In its simplest form, the indexation formula is such that the nominal indexed interest rate at time t, denoted $rind_t$, is equal to the nominal growth rate at time t, denoted g_t.[3]

$$rind_t = g_t \tag{1}$$

[1] This important increase in the debt ratio is also explained by a large recapitalization of the banking system.

[2] Benford, Ostry, and Shiller (2018) provide a recent discussion of the idea.

[3] This paper refers to the nominal interest rate as r, to the nominal indexed interest rate as $rind$, and to the nominal GDP growth rate as g.

Two traditional arguments in favor of such long-term debt instruments are that they 1) enable policymakers to implement more countercyclical fiscal policies, especially during bad times, and 2) reduce the debt variation around its expected value, thus lowering the risk of default.

Regarding debt sustainability, growth-indexed bonds can reduce the variability in the debt ratio (Blanchard et al. 2016, Benford et al. 2016). This reduction in the variance of the debt distribution implies, in turn, a reduction in the default risk and thus in the risk premium. It also implies an increase in the available fiscal space—defined as the difference between the current debt level and a theoretical debt limit, i.e. the threshold above which a country can meet its debt service obligations only by an extraordinary fiscal effort (Pienkowski 2017, Barr et al. 2014).

Regarding the relation to fiscal policy, growth-indexed bonds can foster the implementation of countercyclical fiscal policies by making interest payments procyclical. In fact, GIBs, by reducing the need to implement a fiscal tightening that would endogenously impact the activity during a bust, can avoid the risks of self-defeating fiscal consolidations (Borensztein and Mauro 2004). Conversely, rising debt payments during booms would discipline government expenditures (or force governments to raise taxes) and limit overheating. Growth-indexed bonds, if carefully designed and mostly held by nonresidents, could increase the risk sharing between countries and help smooth the economic cycle. However, if residents hold such indexed bonds, their interest revenue will increase during good times and decrease during bad times. This procyclicality in their income may reinforce, rather than smooth, the economic cycle if not more than offset by an appropriate and well-targeted countercyclical fiscal policy.[4]

Notwithstanding their theoretical benefits, growth-indexed bonds have rarely been issued in practice. Most critics point to the fact that governments would

[4]Such countercyclical fiscal policy would have to target agents whose marginal propensity to consume is higher than bondholders' marginal propensity to consume.

have lower incentives to pursue growth-oriented policies (a moral hazard issue) and that relevant statistics may be manipulated or published with delays making the instrument irrelevant (technical issues). This paper focuses more specifically on the impact of growth-indexed bonds on the upper-tail of the debt distribution and suggests three potential explanations for the low use of GIBs:

(1) simple, standardized GIBs do not provide enough benefits in terms of debt variance reduction;

(2) optimal alternative indexation formulas, which achieve a higher reduction in the debt variance, are too complex and heterogeneous among countries to be issued at a large scale; or

(3) the implicit premium required by risk-averse investors is too high to justify the gains from GIBs.

This paper explores the three potential explanations and provides empirical support for this sovereign noncontingency puzzle. The analysis developed in this paper, which builds on Blanchard et al. (2016), extends the literature in three ways. First, it formally derives the reduction in the variance of the debt-to-GDP ratio associated with the issuance of different types of growth-indexed bonds. In particular, the paper derives the indexation coefficients that would minimize the debt-to-GDP variance for each type of growth-indexed bond. Second, it estimates the reduction in the variance of the debt distribution for 32 advanced countries and emerging markets. Third, it estimates the maximum potential premium that would make growth-indexed bonds too costly to reduce the upper tail of the debt distribution.

The two main conclusions can be summarized as follows.

First, the reduction in the variance of the debt distribution varies importantly across countries and indexation formulas: There is no one-size-fits-all solution. Simple nominal growth-indexed bonds could substantially lower the upper tail of the debt distribution for a few countries like Lebanon or Egypt. However, for most countries, the reduction is less important. For example, if the United States converts all its debt into simple GIBs, the value of the 99th percentile of the indexed debt distribution after 10 years is equal to the value of the 93rd percentile of the nonindexed debt distribution. Put another way, if there is a 7 percent probability that the debt-to-GDP ratio will reach a level higher than a given threshold after 10 years, this probability drops to less than 1 percent if all debt is composed of simple GIBs. This relatively small impact is explained by the fact that simple nominal GIBs offer no protection against cyclical fluctuations in the primary balance, which are quite substantial for some countries. Interestingly, alternative GIBs using an indexation coefficient greater than 1 may bring substantial additional reductions in the upper tail, especially if the sovereign commits to increase the correlation between its growth rate and its primary balance surplus. However, the marginal gains from indexation to both the growth rate and the output gap may not justify the use of such complex indexation schemes.

Second, the share of growth-indexed debt and the size of the potential premium are crucial to the case for issuing GIBs. Indexing up to 20 percent of the stock of debt to the growth rate—a proportion similar to the share of inflation-indexed bonds in some countries—provides almost no benefits in terms of reduction in the upper tail of the debt distribution. Moreover, a net sustained premium of 100 basis points, often discussed in the literature, would actually increase the upper tail of the debt distribution for most countries in the sample and thus limit the case for GIBs.

This paper is organized as follows. Section 2 derives the reduction in the variance of the debt-to-GDP ratio obtained from the issuance of simple nominal GIBs. Section 3 derives the additional reduction in variance from alternative indexation formulas. Section 4 provides empirical estimates of the reduction in the variance of the debt-to-GDP ratio based on Monte Carlo simulations. Section 5 introduces risk-averse bondholders and quantifies the maximum premium that would equalize the value of the upper tail of the debt distribution in the indexed and nonindexed scenarios. Section 6 concludes.

2 Simple GIBs and Debt Distribution

Assessing whether a country's debt is sustainable is derived from two factors: the expected path of the debt-to-GDP ratio and the distribution around it. A public debt is considered sustainable if there is a high probability that the debt-to-GDP ratio follows a stable or downward path going forward. In particular, sovereigns and investors may be worried by the upper tail of the debt-to-GDP ratio distribution. Growth-indexed bonds can provide benefits by reducing the variance of the debt distribution, but these benefits may come at the cost of a higher expected debt ratio if risk-averse investors require a premium to hold them.

Consider the case of an indebted and risk-averse sovereign that issues one-period plain vanilla debt. The debt is rolled over every period and is denominated in local currency. The sovereign prefers to avoid costly default or sharp fiscal adjustment and thus prefers debt ratio levels that are decreasing. Moreover, the sovereign is risk averse and thus prefers low variance in the debt-to-GDP ratio, i.e. low uncertainty regarding the future value of the ratio. Thus the utility function of the sovereign is decreasing in the debt-to-GDP ratio and concave.[5]

This section analyzes the impact of the issuance of long-term growth-indexed bonds, with a multiple-period maturity, on the variance of the debt-to-GDP ratio. Derivations of the formulas are presented in appendix 1, along with an analysis of one-period growth-indexed bonds. It is assumed that bondholder investors are risk neutral and that contracts are costless to write and enforce. For simplicity, it is also assumed without loss of generality that the debt-to-GDP ratio is expected to remain constant. Moreover, the variance-covariance matrix of the nominal growth

[5]This is a simple and restrictive specification. A more general utility function for the sovereign would include output or deviations to output. However, in this setup, a reduction of the debt variance is achieved through lower payments during downturns (and conversely higher payments during upturns), which is consistent with the objective of output stabilization.

rate, g_t, the implicit nominal interest rate on outstanding debt, r_t, and the primary balance as a share of GDP, pb_t, are assumed to be known and given.

Debt Dynamics with Plain-Vanilla Bonds

In order to assess the benefits from growth indexation, the determinants of the variability in the debt-to-GDP ratio are analyzed first. The central government debt level at time t, denoted by D_t, is equal to the debt level at time $t-1$ plus the corresponding nominal interest paid on the outstanding debt $r_t D_{t-1}$ minus the primary balance surplus PB_t:

$$D_t = (1 + r_t)D_{t-1} - PB_t \tag{2}$$

Assuming that the nominal interest rate r_t and the nominal growth rate g_t are relatively small numbers, the well-known equation driving the debt-to-GDP ratio dynamics is approximately equivalent to:

$$\Delta d_t = (r_t - g_t)d_{t-1} - pb_t \tag{3}$$

The evolution of the sovereign debt-to-GDP ratio between $t-1$ and t (Δd_t) is determined by four factors:

- the debt-to-GDP ratio at time $t-1$, d_{t-1};
- the nominal growth rate of GDP at time t, g_t;
- the nominal effective interest rate on outstanding debt at time t, r_t; and
- the primary balance (surplus) as a share of GDP at time t, pb_t.

Consider the case where the expected change in the public debt ratio is equal to zero. The variance in the change in the public debt-to-GDP ratio is equal to:

$$var(\Delta d_t) = var(pb) + d_{t-1}^2 var(r-g) - 2d_{t-1}cov(pb, r-g) \tag{4}$$

Thus the variance of the debt-to-GDP ratio at any time t is determined by the debt ratio in the previous period and the covariance matrix of the nominal growth rate, the nominal effective interest rate, and the primary balance as a share of GDP. Put another way, the higher the variance of the primary balance, the higher the variance of the interest-growth differential, and the more negative the correlation between the primary balance and the interest-growth differential, then the higher the uncertainty in the future public debt ratio.

Debt Dynamics with Simple GIBs

One of the most discussed indexation formulas in the literature is a simple indexation to the nominal growth rate[6] (Borensztein and Mauro 2004, Chamon and Mauro 2006, Blanchard et al. 2016). Consider that a share X of the stock of debt is composed of simple growth-indexed bonds. The indexation formula is specified by including a constant k such that the risk-neutral bondholder is indifferent between rolling over one-period fixed interest bonds or holding a multiple-period GIB until maturity:

$$rind_t = g_t + k \tag{5}$$

Note that k is different from a default, risk, liquidity, or novelty premium. In

[6]Another form of indexation often discussed in the literature, and inspired by the seminal work of Shiller (1993), consists of a bond with a time-varying principal and a coupon indexed on GDP level. Although an indexation to the growth rate is different from an indexation to the GDP level, the two types of bonds are very similar from a solvency point of view, as they both protect the sovereign against growth shocks. However, the coupon-indexed-bond seems preferable from a market liquidity point of view. It provides fiscal relief to the sovereign during a recession via a much lower (potentially negative) coupon, while the variation in the coupon with the principal-indexed version is very small. Additionally, coupon-indexed bonds also make it easier to analyze the impact of alternative indexation formulas on the variance of the debt ratio. Thus, this paper focuses on this specification of the indexation rather than the principal-indexed bond. Williamson (2018, forthcoming) provides a comprehensive comparison of the two different types of indexed bonds.

this case, the equation driving the debt-to-GDP ratio dynamics is:

$$\Delta d_t = [Xk + (1-X)(r_t - g_t)]d_{t-1} - pb_t \tag{6}$$

Moreover, the variance of the debt-to-GDP ratio is:

$$var(\Delta d_t) = var(pb) + (1-X)^2 d_{t-1}^2 var(r-g) - 2(1-X)d_{t-1}cov(pb, r-g) \tag{7}$$

Note that if $X = 1$, the sovereign is fully hedged against shocks to the growth rate and the interest rate. The variance of the debt-to-GDP ratio collapses to the variance of the primary balance as a share of GDP:

$$var(\Delta d_t) = var(pb) \tag{8}$$

Second-order Stochastic Dominance

As k is defined such that the risk-neutral investor is indifferent between rolling over one-period fixed interest bonds or holding a multiple-period GIB until maturity, the expected value of the debt-to-GDP ratio at the end of the considered period is the same for all values of $X \in [0,1]$. Moreover, if the issuance of a share X of simple growth-indexed bonds reduces the variance of the change in the debt-to-GDP ratio compared to the nonindexed case, then this issuance is preferred by the risk-averse sovereign. In this case, the issuance of GIBs is preferred by second-order stochastic dominance.

Does indexation to the growth rate always reduce the variance of changes in the debt ratio? Not necessarily. Using equations (4) and (7), the issuance of a share X of simple growth-indexed bonds is preferred to the nonindexed case by the sovereign by second-order stochastic dominance if:

$$X < 2 - \frac{2cov(pb, r-g)}{d_{t-1}var(r-g)} \tag{9}$$

Note that the share X can be as high as 1 if the following condition holds:

$$1 - \frac{2cov(pb, r-g)}{d_{t-1}var(r-g)} > 0 \qquad (10)$$

Consider the case of an indebted sovereign, where $d_{t-1} > 0$. Thus a sufficient condition can be stated as follows: Converting all the stock of debt into simple long-term growth-indexed bonds is preferred by second-order stochastic dominance if the interest-growth differential and the primary balance as a share of GDP are negatively correlated. Using data from the International Monetary Fund's (IMF) April 2017 *World Economic Outlook* (WEO), the correlation between the primary balance and the interest-growth differential is found to be negative for a large majority of countries (143 over 181) from 1996 to 2016, but not for all countries. The issuance of long-term growth-indexed bonds can help countries stabilize their debt-to-GDP ratio by reducing the negative correlation between the interest-growth differential and the primary balance. This correlation is equal to zero if all the stock of debt is composed of growth-indexed bonds.

The optimal share $X^\star$ that achieves the lowest variance in the debt-to-GDP ratio is given by the first-order conditions of the minimization of the variance of changes in the debt ratio with respect to X:

$$X^\star = 1 - \frac{cov(pb, r-g)}{d_{t-1}var(r-g)} \qquad (11)$$

Note that if the covariance between the interest-growth differential and the primary balance as a share of GDP is negative, then $X^\star$ would be higher than 1. This means that the sovereign would prefer to be even further hedged against growth rate fluctuations. This is explored in the next section.

The two obvious advantages of this formula using a one-to-one indexation to the nominal growth rate is that it is extremely simple, especially when compared to the formulas used during the few cases indexed bonds were issued,[7] and offers

[7] See M. L. Anthony, N. Balta, T. Best, S. Nadeem, and E. Togo, "What history tells us about state-contingent debt instruments," VoxEU.org, June 6, 2017.

standardization among countries, which could facilitate its broad issuance by different countries and acceptance by the markets.

However, it does not offer protection to the sovereign against adverse shocks to the primary balance. As the primary balance is generally positively correlated with the cycle, through the presence of fiscal stabilizers, public finances deteriorate during bad times. Actually, the impact of lower growth through the deterioration of the primary balance has played a substantial role in the recent increase in public debt levels in advanced economies (Mauro and Zilinsky 2016).

In principle, it is possible for the sovereign to reduce the variance of its debt ratio distribution to a level lower than the variance of the primary balance. The next section analyzes the impact of alternative indexation formulas, namely weighted combinations of the growth rate and the output gap, on the variance of the debt-to-GDP ratio.

3 Debt Dynamics with Alternative Indexation Formulas

This section explores alternative indexation formulas, more complex than the simple growth-indexed bond presented above, but potentially preferred by the sovereign by second-order stochastic dominance.

A. A Benchmark: The Fully Contingent Formula

The fully contingent formula is the indexation formula that would eliminate the uncertainty about the future path of the debt-to-GDP ratio. Solving equation (3) under $\Delta d_t = 0$ gives:

$$rind_t = \frac{pb_t}{d_{t-1}} + g_t \qquad (12)$$

This indexation formula achieves the highest reduction in the variance of the debt ratio. In fact, the variance is equal to zero. This formula provides an important insight: The optimal formula indexes the interest rate not only to the growth rate, but also to the primary balance and the lagged debt-to-GDP ratio. However, such a formula has an obvious shortcoming: Investors will not accept a financial return based on a variable that the government can arbitrarily alter. Indeed, a sovereign would have strong incentives to misbehave, as a decrease in the primary balance would decrease interest payments, creating an obvious moral hazard issue. This can be seen as a dissuasive fixed cost to include in the price of the bond, and thus it is preferable for the government to use a bond with an alternative indexation scheme that is more acceptable to investors.

The next subsections introduce alternative indexation formulas, to a weighted combination of the nominal growth rate and the output gap. The primary balance can be decomposed between the cyclically adjusted primary balance and the cyclical primary balance (the part of the primary balance that automatically reacts to the cycle). Absent discretionary fiscal measures, the primary balance would respond entirely to cyclical components. This suggests that, apart from growth, both parties could accept an indexation to another variable describing the state of the economy while being exogenous to the government, such as the output gap.[8]

In addition to output, fiscal variables may respond to changes in asset prices, real estate prices, interest rates, exchange rates, commodity prices, and other variables. The alternative formulas could take various forms, ideally reflecting the sovereign's capacity to serve its debt, but this paper keeps them relatively simple and restricts the analysis to two variables only: the growth rate and the output gap. The main reasoning would be the same if using any other variable.

[8] This paper refers to any case where output is above its potential as a positive output gap. Conversely, a negative output gap means that output is below its potential.

B. Growth-indexed Bonds

In the general case of a growth-indexed bond, the indexation formula is:

$$rind_t = cg_t + k \tag{13}$$

Where c is the indexation coefficient, or the elasticity of the return to the growth rate. In the simplest case analyzed above $c = 1$, but in theory this coefficient could take any positive value. Again, the constant k is defined such that the risk-neutral bondholder is indifferent between rolling over one-period fixed interest bonds or holding a multiple-period GIB until maturity.

The debt dynamics equation becomes:

$$\Delta d_t = [Xk + (1 - X)r_t - (1 - cX)g_t]d_{t-1} - pb_t \tag{14}$$

The optimal indexation coefficient $c^\star$ is given by the first–order conditions of the minimization of the variance of changes in the debt ratio with respect to c:

$$c^\star = \frac{1}{X} + \frac{cov(g, pb)}{Xd_{t-1}var(g)} - \frac{(1 - X)cov(r, g)}{Xvar(g)} \tag{15}$$

Note that if $X = 1$, the optimal indexation coefficient $c^\star$ simplifies to:

$$c^\star = 1 + \frac{cov(g, pb)}{d_{t-1}var(g)} \tag{16}$$

Moreover, the variance using this optimal coefficient $c^\star$ if $X = 1$ is:

$$var(\Delta d_t) = var(pb)(1 - (\rho_{g,pb})^2) \tag{17}$$

Where $\rho_{g,pb}$ is the correlation between the nominal growth rate and the primary balance as a share of GDP. Thus the efficiency of the optimal indexation coefficient $c^\star$ depends on the correlation between the nominal growth rate and the primary balance. If the nominal growth rate and the primary balance are uncorrelated, then we are back to the simple growth-indexed bonds case, and the minimum variance achievable by GIBs is equal to the variance of the primary balance. If the nominal growth rate and the primary balance are perfectly correlated, then the variance is equal to zero and we are back to the fully contingent formula case.

If all the debt is indexed to the growth rate, then the deviation from $c^{\star} = 1$ depends on how the primary balance varies with the growth rate. An increase in the absolute value of the correlation between the primary balance and the growth rate makes the sovereign better able to control the debt distribution through the issuance of such bonds, as the growth rate is a better proxy for the primary balance. This gives a simple characterization of the optimal degree of indexing chosen: It is such that variations of debt are uncorrelated with the variable used in the indexation formula (Blanchard 1979).

C. Growth-gap Indexed Bonds

In the case of a long-term growth-gap indexed bond, the formula is:

$$rind_t = ag_t + bz_t + k \tag{18}$$

Where z_t is the output gap at time t, a is the indexation coefficient to the growth rate, and b the indexation coefficient to the output gap. The output gap is defined as the difference, in percentage points, between current and potential output. Again, k is a constant such that the risk-neutral investor is indifferent between holding this long-term growth-gap indexed bond until maturity and rolling over one-period nonindexed bonds over the same period.

The debt dynamics equation becomes:

$$\Delta d_t = [X((a-1)g_t + bz_t + k) + (1-X)(r_t - g_t)]d_{t-1} - pb_t \tag{19}$$

Moreover, if $X = 1$, the optimal indexation coefficients $a^{\star}$ and $b^{\star}$ satisfy the following conditions:

$$a^{\star} = 1 + \frac{cov(pb,g)var(z) - cov(pb,z)cov(z,g)}{d_{t-1}(var(g)var(z) - cov(z,g)^2)} \tag{20}$$

$$b^{\star} = \frac{cov(pb,z)var(g) - cov(pb,g)cov(z,g)}{d_{t-1}(var(g)var(z) - cov(z,g)^2)} \tag{21}$$

Under which conditions would this formula dominate the previous one? Two factors are at play. First, if the covariance between the growth rate and the output gap is high, excluding the output gap will not be very costly in terms of reduction in the debt variance, while it will reduce the costs associated with its inclusion in the formula. If the correlation between the growth rate and the output gap is equal to zero, the potential gains from the second formula compared to the first are maximized. Second, the gains are higher if the covariance between the primary balance and the output gap is important.

More formally, if the correlation between the growth rate and the output gap is equal to zero, then the variance using optimal coefficients $a^{\star}$ and $b^{\star}$ is:

$$var(\Delta d_t) = var(pb)(1 - (\rho_{g,pb})^2 - (\rho_{z,pb})^2) \tag{22}$$

Where $\rho_{g,pb}$ is the correlation between the nominal growth rate and the primary balance as a share of GDP and $\rho_{z,pb}$ is the correlation between the output gap and the primary balance as a share of GDP.

Moreover, note that $b^{\star} = 0$ if $cov(pb,z)var(g) = cov(pb,g)cov(z,g)$, i.e. if:

$$\rho_{pb,z} = \rho_{pb,g}\rho_{g,z} \tag{23}$$

If the correlation coefficient between the primary balance and the output gap is equal to the correlation between the primary balance and the growth rate times the correlation between the growth rate and the output gap, then indexing on the output gap is useless against changes in the primary balance. In this case, the growth-indexed bond achieves the lowest variance in the debt ratio.

Similarly, note that $a^\star = 1$ if $cov(pb, g)var(z) = cov(pb, z)cov(z, g)$, i.e. if:

$$\rho_{pb,g} = \rho_{pb,z}\rho_{g,z} \tag{24}$$

If the correlation coefficient between the primary balance and the growth rate is equal to the correlation between the primary balance and the output gap times the correlation between the growth rate and the output gap, then indexing on the growth rate is useless against changes in the primary balance.

Combining (23) and (24) obtains: $|\rho_{g,z}| = 1$ and $\rho_{pb,g} = \rho_{pb,z} = 0$

In this case, simple growth-indexed bonds achieve the lowest variance in the debt ratio. Thus the choice between issuing GIBs or not, the form of the GIB and the optimal indexation coefficients, are endogenous to the behavior of the relevant variables and depend on the joint distribution of the interest rate, the growth rate, the primary balance, and any variable included in the indexation formula (here, the output gap). In particular, the optimal indexation coefficients depend on the relation of the primary balance to the variables included in the indexation formula.

The next section estimates the impact of different types of growth-indexed bonds on the variance of the debt distribution based on Monte Carlo simulations.

4 Simulations

A. Methodology and Data

In order to quantitatively assess the gains—defined as the reduction in the upper-tail of the debt-to-GDP ratio distribution—obtained from using each indexation formula, the fan-chart approach used in Blanchard, Mauro and Acalin (2016) is expanded. The analysis for growth-indexed bonds covers 32 advanced and emerging economies.[9] The annual data come from the April 2017 IMF WEO database and cover the period from 1996 to 2016. For all countries, the data used are the nominal GDP growth rate, the effective annual nominal interest rate on debt (constructed as the ratio of the difference between the primary balance and the overall fiscal balance over the lagged debt level[10]), and the primary balance as a share of GDP. Throughout the simulation period, the expected values of the interest rate, the growth rate and the ratio of the primary balance to GDP, are taken to be equal to the April 2017 IMF WEO forecasts up to 2022, and are fixed at the 2022 level from then on. This defines the baseline scenario for the evolution of the debt-to-GDP ratio.

Regarding the distribution of realizations around this baseline, the distribution of shocks for r, g, and pb are assumed to be a multivariate normal distribution, with a covariance matrix given by the empirical covariance matrix estimated over 1996–2016. The shocks are assumed to be independently and identically distributed over time, that is, shocks occurring this year have no implications for the distribution of the shocks next year.[11] Monte Carlo simulations for 10-year and 20-year periods are then generated through 10,000 random draws from the multivariate distribution.

The same methodology is replicated for a subgroup of 14 countries for which data are also available on both the output gap from the IMF and on the effective annual nominal interest rate on debt (constructed as the ratio of actual nominal interest payments over the lagged debt level) from the European Commission's annual macroeconomic (AMECO) database. Whenever no forecast for the output

[9] The countries are: Argentina, Australia, Austria, Belgium, Brazil, Cameroon, Canada, Chile, Colombia, Costa Rica, Egypt, France, Germany, Greece, India, Indonesia, Israel, Italy, Japan, Korea, Lebanon, Malta, Mexico, the Netherlands, Peru, Portugal, South Africa, Spain, Sweden, Turkey, the United Kingdom, and the United States.

[10] This is only an approximation as this corresponds to interests paid minus interests received over the lagged debt level.

[11] Estimates from a vector auto-regression with one lag, allowing shocks to be serially correlated over time, are reported in appendix 2. The results are very similar in both cases.

gap is available for one of the 14 countries, the output gap in the baseline scenario is assumed to follow an autoregressive process with an AR(1) coefficient equal to 0.9. No forecasts are available after 2018 for the interest rate from AMECO. Thus a regression of the interest rate from AMECO is fitted on the approximation obtained by using IMF data, and this linear regression and IMF forecasts are used to compute the baseline scenario for the AMECO interest rate.[12] Appendix 1 provides more details on the methodology and underlying assumptions.

B. Simple Nominal Growth-indexed Bonds

As discussed in the previous part, the gains from indexation crucially depend on the covariance matrix of the growth rate, the interest rate, and the primary balance (table 1). For all but three countries in the sample (Belgium, Korea, and Argentina), the interest-growth differential and the primary balance are negatively correlated, suggesting that they would benefit from a simple indexation to the growth rate.

However, the gains from indexation may vary from one country to another. Countries with a high debt-to-GDP ratio and/or a volatile interest-growth differential, such as Lebanon, Spain, and Argentina, are more likely to benefit from indexation. On the other hand, countries with a less volatile interest-growth differential, a more volatile primary balance, and/or a low debt ratio, such as the United Kingdom or Cameroon, are less likely to benefit from an indexation to the growth rate.

The impact from indexation on the reduction in the upper-tail of the debt distribution after 10 years is gauged in two steps. First, simulate the evolution of the debt-to-GDP ratio under nominal bonds and nominal growth-indexed bonds in the cases where 100 percent and only 20 percent of the stock of debt is indexed to growth. The fan charts in figure 2 report the debt-to-GDP levels corresponding

[12]The R^2 statistics from a regression of the AMECO interest rate on the IMF approximation is higher than 0.85 for all 14 countries in the subsample (see appendix 1).

to the 1st, 50th, and 99th percentiles of the different distributions for selected countries. By construction, the 50th percentile (the baseline) is the same under indexation and nonindexation.[13]

Second, take the value of the debt-to-GDP ratio corresponding to the 99th percentile of the distribution with growth-indexed bonds, and find the percentile of the distribution of nonindexed debt corresponding to this value. This allows the reduction in the upper-tail of the debt distribution to be quantified: The lower the percentile, the higher the reduction in the upper tail of the distribution. The numbers are reported in table 2.

For example, in the case of Lebanon, the value of the 99th percentile of the debt distribution with growth-indexed bonds is equal to the value of the 65th percentile of the debt distribution without indexation. Thus, Lebanon could virtually avoid about 34 percent of the worst outcomes in its debt ratio in the future by converting all of its stock of debt into nominal growth-indexed bonds. On the opposite, for Cameroon, the value of the 99th percentile of the debt distribution with growth-indexed bonds is equal to the value of the 98th percentile of the debt distribution without indexation. There would be almost no reduction in the upper tail of the distribution.

Most countries in the sample could virtually avoid about less than 10 percent of the worst outcomes in their debt ratio in the future by converting all their stock of debt into nominal growth-indexed bonds. The vector autoregressive specification provides very similar results, except for a few countries. In particular, the gains from indexation are higher for Chile and Australia using the autoregressive specification, while they are lower for Mexico and Argentina. More strikingly, in

[13]To be explicit, the constant k in the different indexation formulas is such that the expected return on a long-term GIB is equal to the expected return from rolling over one-period nominal bonds over the same period. For simplicity, this constant is equal to the expected implicit interest rate minus the expected GDP growth rate for each year in the simulation. It is thus not "constant" over time. However, as k is predetermined at issuance and is not revised afterwards, it does not contribute to the variance of the indexed interest rate and hence to the variance of the debt-to-GDP ratio.

both cases, and for all countries in the sample, an indexation of only 20 percent of the stock of debt to the growth rate would provide almost no reduction in the upper tail of the distribution.

C. Growth-indexed Bonds with Optimal Coefficient

The same analysis for the two alternative formulas with optimal coefficients is replicated. First, the optimal indexation coefficients, given by the formulas described in the previous part and that depend on the relation of the primary surplus to the variables included in the indexation formula, are computed.

The next table (table 3) shows that if a country decides to issue a debt entirely indexed to its growth rate, the optimal $c^\star$ coefficient would be quite different from 1 for most countries. For all countries in the sample except Argentina, the coefficient would be higher than 1, as suggested by the positive correlation between the growth rate and the primary balance over the estimated period. However, the optimal coefficient varies from one country to another. The coefficient ranges from 1.04 for Lebanon, where the correlation between the growth rate and the primary balance is low (0.16) and the debt-to-GDP ratio is high (143 percent), to 3.08 for Chile, which is an outlier. Most values range between 1.10 and 2.10.

It is important to note that the computed optimal coefficients do not give any indication regarding their efficiency in reducing the support of the debt distribution. This efficiency, as shown in equations (17) and (22), is determined by the relation of the primary balance to the variables included in the indexation formula. The more effectively a variable can be used as a proxy of the primary balance, the higher the gains from indexation with optimal coefficients. There are important differences among the R^2 statistics, which are low in most cases, in the regressions of the primary balance on the growth rate and the output gap (table 4). For example, one may expect growth-gap indexation to provide more benefits to the United States than to Malta, as the growth rate and the output gap explain more of the variability of the primary balance in the former country than in the latter.

Indexation to the growth rate, using the previously computed optimal coefficients, would provide more benefits to most countries, but the additional gains compared to the simple case (with the coefficient c equal to 1) vary considerably from one country to another (table 5). Figure 3 shows the different paths for selected countries.

D. Growth-gap-indexed Bonds with Optimal Coefficients

For a subsample of 14 advanced economies, the optimal indexation coefficients on the growth rate, $a^\star$, and on the output gap, $b^\star$, can be computed using additional available data on the output gap from the IMF and the effective interest rate from AMECO (table 6).

If a country decides to issue a debt entirely indexed to both its growth rate and its output gap, the optimal coefficient $a^\star$ would range from 0.80 to 2.27, while the optimal coefficient $b^\star$ would range from -0.27 to 1.23. The $a^\star$ coefficient for Germany, less than 1, suggests that once the effect of the output gap is taken into account, the primary balance is negatively correlated with the growth rate, as suggested by the regressions of the primary balance on the growth rate and the output gap. The negative $b^\star$ coefficient for Italy, Greece, Portugal, and Spain is explained by the negative beta coefficient on the output gap in the primary balance regression. This is due to the fact that those countries had to tighten fiscal policy while output was below potential—an example of the vicious cycle described in the introduction.

For most countries in the subsample, there is a strong positive correlation between the growth rate and the output gap, equal to 0.6 on average (table 7), suggesting that an indexation to both the output gap and the growth rate may bring only a few additional benefits compared to an indexation to the growth rate only.

This indexation formula dominates the previous formulas, but again the gains vary importantly across countries (table 8). As expected, the gains compared to the growth-indexed bond with the optimal coefficient are irrelevant for Malta, while they are more important for the United States. The gains compared to the growth-indexed bonds with the optimal coefficient are irrelevant for Spain and the Netherlands, as the effect of the output gap on the primary balance is not significant once the impact of the growth rate is taken into account. Despite the fact that the growth rate and the output gap are good explanatory variables for the primary balance in Japan, an indexation to both variables provides only marginal gains compared to the growth-indexed bond. This has a simple explanation: In Japan, given the very high debt ratio, most of the variation in the debt ratio comes from the uncertainty regarding the interest-growth differential, not from the primary balance itself. This indexation would provide some gains for other countries, but the small additional gains may not justify such complex formulas, especially given the uncertainty regarding output gap measurement.

Several conclusions can be drawn from this section. Indexed debt may reduce the variance of the debt-to-GDP ratio. However, the gains vary importantly from one country to another and depend on the comovements between the relevant variables. Indexation to the growth rate with an optimal indexation coefficient may bring some additional benefits in terms of reduction in the upper tail of the debt distribution compared to simple growth-indexed bonds. Alternative indexation schemes to both the growth rate and the output gap may also bring additional benefits, but the marginal gains compared to an indexation to the growth rate with the optimal coefficient may not justify the use of such complex indexation schemes. The introduction of the output gap—which is very difficult to measure and could be substantially revised over time—in the formula would most probably imply a higher premium on indexed bonds. This premium may offset small marginal benefits regarding the reduction in the variance of the debt ratio.

The introduction of an indexation coefficient other than 1 should not present any particular technical difficulty (for example, instead of paying the growth rate for a given period, the bond could pay twice the growth rate) and may bring some additional benefits for most sovereigns, especially if they commit to increase the correlation between the growth rate and the primary balance through fiscal stabilizers. However, it may come at the cost of a higher premium in order to persuade investors to accept returns that are more volatile.

The simulations presented in this section rely on three assumptions. The first assumption is that the joint distribution of the interest rate, the growth rate, the primary balance and the output gap that prevailed in the past will remain the same in the future. However, the covariance matrix and, by implication, the optimal coefficients computed above are likely to evolve with time.

The second assumption is that the sovereign would be able to issue bonds with very specific formulas. Despite this assumption, the marginal gains in terms of reduction in the debt ratio variance resulting from an alternative indexation to both the growth rate and the output gap compared to a restricted indexation to the growth rate with the optimal coefficient may not justify the use of such complex indexation schemes. This is explained by the relatively low R^2 statistics in the regressions of the primary balance on growth and the output gap. A stronger relationship would increase the efficiency of alternative indexed bonds with an optimal coefficient, as growth and the output gap are a better proxy for the primary balance. Thus, by making the primary balance more correlated with the growth rate and/or the output gap, the sovereign would increase the efficiency of alternative indexation formulas. In the extreme case where the R^2 statistic is equal to 1, i.e. if changes in the primary balance are entirely explained by fluctuations in the output, the debt ratio distribution under an alternative indexation formula with optimal coefficients would be degenerate.

The third assumption is that investors are risk neutral and that the different types of growth-indexed bonds do not have to pay any premium relative to the expected interest return on traditional plain-vanilla bonds. This assumption is relaxed in the next section.

5 Introducing the Risk-averse Investor

The previous simulations only focused on the reduction of the variance of the debt distribution, assuming risk-neutral investors and costless contracts. However, the gains procured by each formula have to be compared to the potential cost of issuing the different types of indexed bonds. The potential cost can come from two different factors: the investors' risk aversion and the market structure.

If investors are risk averse, the cost of issuing state-contingent contracts may outweigh the benefits, in particular if the payoffs are highly correlated with investors' income or if volatility in payoffs is high. This may be particularly true of debt indexed on the output gap, which is more persistent than the growth rate. Additionally, difficulties in pricing the instrument due to their novelty (Costa et al. 2008), the complexity of the indexation formula, or the lack of liquidity in the indexed-bond market may raise the cost of issuing such bonds. Conversely, the issuance of indexed debt may reduce the probability of default and thus reduce the default premium on both indexed and nonindexed debt.

If the former effect dominates the latter, then risk-averse investors require that GDP-linked bonds pay a positive, net premium in order to hold them. In this case, the expected value of the debt-to-GDP ratio in the next period is higher with GDP-linked bonds than without indexed debt.

The indexed-interest rate is now denoted by:

$$rind_t = g_t + k + premium \tag{25}$$

Indexed bonds decrease the variance of the distribution of the debt ratio, but if the bonds require a high enough premium for investors to buy and hold them, the benefits of a smaller variance may be more than offset by a higher median of the debt ratio under the baseline. Figure 4 shows the impact of a 100-basis-point net premium on the debt distribution in the indexed case of the United Kingdom. In this example, a sustained premium of 100 basis points over a 10-year period would make indexed debt too costly to provide relevant benefits in terms of debt distribution.

A sovereign with high risk aversion could be willing to pay a positive premium as long as the value of the upper tail of the debt-to-GDP ratio distribution in the indexed case does not exceed the value of the upper tail in the nonindexed case, i.e. as long as extreme upper tail debt ratios are more likely under nonindexation than under indexation.

This section quantifies the maximum premium that an issuing country would be willing to pay to transfer the debt service risk to investors—defined as the premium that equalizes the values of the 95th or 99th percentile of the indexed distribution using simple growth-indexed bonds to that of the nonindexed distribution at the final year of the forecast horizon.[14]

The formula for the maximum premium is the following:

$$Max_{premium} = (\frac{dnonindexed_t}{d_{t0}})^{1/t} - (\frac{dindexed_t}{d_{t0}})^{1/t} \tag{26}$$

Where $dnonindexed_t$ is the value of the debt-to-GDP ratio corresponding to the nth percentile of the distribution in the nonindexed case at time t, $dindexed_t$ is the value of the debt-to-GDP ratio corresponding to the nth percentile of the distribution in the indexed case at time t, d_{t0} is the debt-to-GDP ratio at time t0 (end of 2016) and t is the horizon forecast (10 or 20 years).

Table 9 shows what net premium would make the 95th or 99th percentile of

[14]To clarify, this paper does not attempt to estimate the impact of the issuance on each component of the premium but rather to estimate the novelty/liquidity/risk premium netted by the reduction in the default premium that would make such instruments too costly.

the indexed debt equal to their respective values under nonindexed debt after 10 or 20 years, i.e. the premium that would increase the median forecast such that it cancels the benefits obtained from the reduction in the debt variance.

The higher the gains obtained in the previous simulations and the lower the initial debt level, the higher the threshold on the premium. According to the simulations, a few countries such as Lebanon[15] or Greece would still benefit from indexation, in the sense of a reduction of the upper tail of the distribution, even if the premium is above 100 basis points. However, the premium has a nonlinear effect: As the time horizon increases, the impact of a rise in the baseline tends to dominate the impact of a lower variance. Additionally, if a more conservative approach is taken, the premium that would equalize the 95th percentile of both distributions after 20 years is below 100 basis points for most countries.

Put another way, a sustained net annual insurance premium of about 1 percent of GDP over 10 years would be too costly for advanced economies (where debt-to-GDP ratios are close to 100 percent) to justify the use of such debt instruments.

[15]Note: In the case of Lebanon and Japan, two countries with a very high debt ratio, a premium of 100 basis points would make the 1st percentile of the distribution in the indexed case higher than the 50th percentile of the distribution (baseline) in the nonindexed case.

6 Conclusion

As a consequence of the financial crisis, most advanced economies are currently faced with high debt-to-GDP ratios, while their fiscal and monetary policies are often constrained by institutional arrangements or the zero-lower bound on short-term interest rates. This makes the need for alternative policy options more relevant. The issuance of growth-indexed bonds can provide benefits, but such benefits have to be weighed against potential costs. The main findings of this paper can be summarized as follows.

A restricted indexation to the growth rate, in the form of a growth-indexed bond, reduces the variance of the debt distribution but provides limited benefits, as it does not protect the sovereign against fluctuations in the primary balance, which could be substantial. In order to stabilize the debt ratio during bad times, the sovereign will still have to implement procyclical fiscal policies, although to a lesser extent than it would without indexation. Alternative indexation schemes, to variables such as the output gap, can further reduce the debt distribution, but the gains are quantitatively small and may not justify such complex indexation schemes.

A possibility for some sovereigns, if growth-indexed bonds are successfully issued, would be to issue growth-indexed bonds with different indexation coefficients, in order to have a portfolio of indexed debt with an indexation coefficient close to the optimal coefficient. A country for which the optimal coefficient $c^{\star}$ is close to 2 could for example issue half of its indexed debt as simple growth-indexed bonds and half of its indexed debt as growth-indexed bonds with an indexation coefficient of 3. However, alternative indexation formulas seem too heterogeneous among countries to be issued at a large scale. Another possibility, left for further research, would be to index the principal to the GDP level and to index the coupon to the growth rate. The principal indexation would help stabilize the debt-to-GDP ratio, while the coupon indexation would help implement countercyclical fiscal policies.

Notwithstanding the potential benefits, indexed bonds have never been issued outside a sovereign debt restructuring context. Technical obstacles to the broader use of state-contingent bonds relate to the timing of the payment, the potential misreporting of GDP data by issuers, difficulties in ex post verification of the data, and complications from ex-post GDP revisions and methodological changes. Some issues are also related to the treatment of negative coupons, the risk premium due to the higher uncertainty regarding the net present value of the instrument, or the operational definition of the output gap. Practical issues associated with the issuance of indexed bonds may imply a higher cost to issue them, and this higher cost may outweigh additional benefits: The size of the premium is crucial. A premium of 100 basis points —often discussed in the literature— and even lower for some countries may cancel the potential benefits from indexation. As shown by the empirical estimates, the share of indexed debt also matters: A small share of indexed debt may only provide limited benefits to the country, while it may require a higher premium from market participants (for liquidity reasons, or lack of diversification). This suggests that only a large and/or coordinated action could move the equilibrium from a situation of non-indexed to indexed sovereign debt. It is likely that an initiative by advanced economies to issue indexed bonds would ease their more widespread implementation.

Finally, this paper considers the primary balance and output as exogenously determined. A key follow-up alley of research would be to analyze the endogenous interactions between indexed debt, fiscal policy, and output. As indexed debt reduces the need to tighten fiscal policy during downturns, gains from indexed debt may prove to be more relevant once the impact of fiscal multipliers is taken into account. It may also be useful to explore the relations to monetary policy, in particular in monetary unions, and the financial system in order to fully quantify the impact of indexed debt.

In conclusion, there is still a case for issuing growth-linked bonds in some advanced and emerging countries, but this case has to be nuanced.

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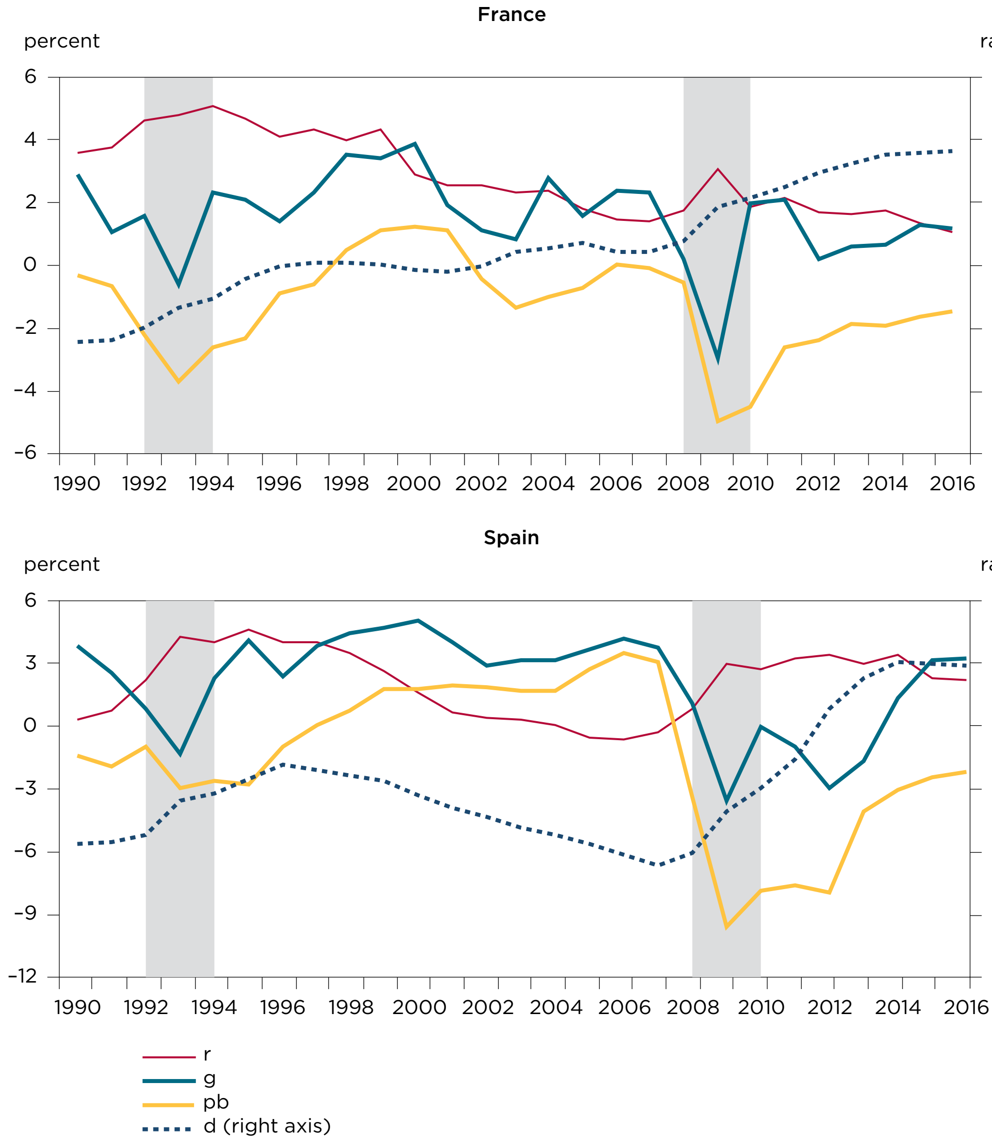
TABLES AND FIGURES

Figure 1 Real interest rate, growth, primary balance and debt ratio in France and Spain (1990–2016)



r = real implicit interest rate (computed as real interest payments over lagged debt level);
g = real GDP growth rate; pb = primary balance (surplus), percent of GDP; d = debt-to-GDP ratio

Note: Shaded areas denote recessions.

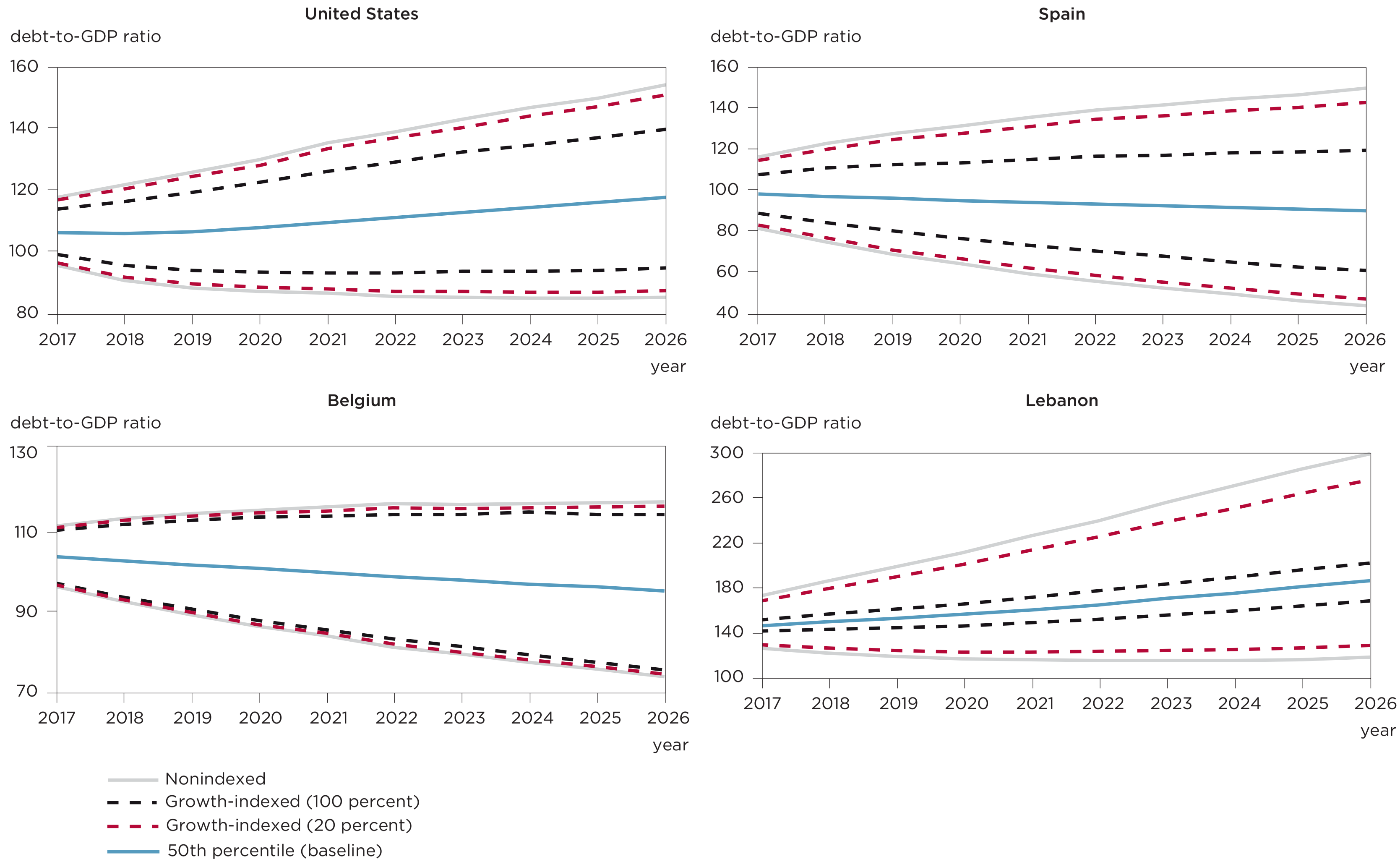
Source: International Monetary Fund *World Economic Outlook* database, April 2017.

Table 1 Descriptive statistics by country (1996–2016)

				Primary balance (surplus) as a share of GDP and difference between nominal implicit interest rate and nominal GDP growth rate		Primary balance (surplus) as a share of GDP and nominal GDP growth rate	
	Debt-to-GDP ratio, 2016	Variance of difference between nominal implicit interest rate and nominal GDP growth rate	Variance of primary balance (surplus) as a share of GDP	Correlation	Covariance	Correlation	Covariance
Country	**d_2016**	**var(r–g)**	**var(pb)**	**corr(pb, r–g)**	**cov(pb, r–g)**	**corr(pb, g)**	**cov(pb, g)**
Argentina	51.27	230.22	6.38	0.15	5.68	–0.20	–6.49
Australia	41.09	15.65	4.66	–0.24	–2.01	0.44	2.06
Austria	83.87	2.51	1.60	–0.22	–0.45	0.32	0.64
Belgium	105.53	2.81	7.50	0.08	0.38	0.55	2.52
Brazil	78.32	18.06	4.02	–0.76	–6.52	0.82	6.15
Cameroon	32.82	3.11	70.53	–0.15	–2.24	0.07	0.81
Canada	92.33	9.41	7.65	–0.06	–0.47	0.48	3.86
Chile	21.18	21.26	9.16	–0.64	–8.95	0.63	8.23
Colombia	47.59	11.65	1.58	–0.16	–0.68	0.01	0.07
Costa Rica	43.71	9.73	4.27	–0.74	–4.91	0.87	8.50
Egypt	97.07	17.34	1.13	–0.72	–2.63	0.66	2.66
France	96.65	2.32	2.55	–0.47	–1.14	0.75	2.10
Germany	67.65	6.12	2.31	–0.44	–1.65	0.35	0.99
Greece	181.33	22.65	10.42	–0.22	–3.35	0.32	6.31
India	69.54	9.38	2.92	–0.22	–1.13	0.31	1.61
Indonesia	27.85	24.39	2.09	–0.53	–4.18	0.24	3.15
Israel	62.21	6.74	2.75	–0.11	–0.46	0.46	1.71
Italy	132.60	3.26	2.81	–0.13	–0.40	0.59	2.27
Japan	239.18	4.28	4.44	–0.50	–2.18	0.48	2.07
Korea	38.55	17.24	2.51	0.38	2.48	0.58	2.87
Lebanon	143.42	34.65	19.41	–0.26	–2.93	–0.14	–3.77
Malta	59.42	12.31	2.95	–0.20	–1.22	0.07	0.42
Mexico	58.10	13.12	1.93	–0.42	–2.02	0.59	6.90
Netherlands	62.57	3.48	5.51	–0.72	–3.17	0.91	5.70
Peru	24.82	13.77	3.29	–0.55	–3.76	0.61	4.11
Portugal	130.30	7.59	5.28	–0.33	–2.09	0.37	2.85
South Africa	50.47	5.56	4.49	–0.54	–2.68	0.76	4.73
Spain	99.26	12.30	16.04	–0.93	–13.07	0.96	15.03
Sweden	41.65	6.48	3.67	–0.04	–0.18	0.46	1.93
Turkey	29.10	37.71	3.80	–0.12	–1.37	0.65	18.51
United Kingdom	89.16	2.89	8.99	–0.38	–1.93	0.63	3.64
United States	107.35	3.85	10.87	–0.62	–3.77	0.60	3.88

Sources: International Monetary Fund *World Economic Outlook* database, April 2017; and author's calculations.

Figure 2 Debt-to-GDP ratio forecasts for different shares of growth-indexed debt



Note: The lines represent the 1st and 99th percentiles of the debt-to-GDP distribution for debt that is nonindexed debt (grey), all growth-indexed (black), and 20 percent growth-indexed (red). By construction the 50th percentile (baseline) is the same for all cases (blue).

Source: Author's calculations.

Table 2 Percentile of the distribution of nonindexed debt corresponding to the 99th percentile of the distribution of growth-indexed bonds

Country	100 percent indexed debt	20 percent indexed debt
Lebanon	65	97
Egypt	74	97
Greece	75	97
Japan	78	97
Argentina	80	97
Brazil	81	97
Mexico	84	97
Portugal	85	97
Italy	89	98
Spain	89	98
Costa Rica	90	98
Indonesia	90	98
France	91	98
Germany	91	98
Colombia	91	98
India	91	98
United States	92	98
Austria	92	98
Malta	92	98
Israel	92	98
Canada	94	98
Turkey	94	98
South Africa	94	98
Peru	94	98
Netherlands	95	98
Australia	95	98
Chile	95	98
United Kingdom	96	98
Belgium	97	98
Sweden	97	98
Korea	98	98
Cameroon	98	98

Note: Indexation to the growth rate only. The numbers represent the percentile of the nonindexed distribution corresponding to the 99th percentile of the indexed distribution after 10 years of independent and identically distributed (iid) shocks.

Source: Author's calculations.

Table 3 Optimal indexation coefficient to the nominal growth rate, when all debt is indexed

Country	Optimal indexation coefficient (c^*)
Argentina	0.92
Lebanon	1.04
Colombia	1.05
Malta	1.06
Greece	1.09
Egypt	1.14
Mexico	1.18
Portugal	1.19
Japan	1.21
India	1.25
Austria	1.30
Italy	1.32
Turkey	1.36
Germany	1.42
Israel	1.45
Canada	1.50
Brazil	1.57
Indonesia	1.62
France	1.71
Korea	1.76
Belgium	1.86
Costa Rica	1.88
Sweden	1.96
United States	1.97
Spain	1.99
Australia	2.05
South Africa	2.07
United Kingdom	2.09
Peru	2.19
Cameroon	2.26
Netherlands	2.28
Chile	3.08

Note: In the c^* formula, dt–1 is fixed to the debt-to-GDP ratio in 2016 in order to have a fixed (nontime varying) coefficient. Although this tends to decrease the efficiency of the optimal coefficient as the debt ratio deviates from its initial level, this effect is limited.

Source: Author's calculations.

Table 4 Regression of the primary balance on the nominal growth and the output gap (R^2 statistics)

Advanced economies

	Growth rate (g)	Growth rate and output gap (g,z)
Australia	0.19	0.49
Austria	0.10	0.23
Belgium	0.30	0.31
Canada	0.23	0.33
France	0.57	0.60
Germany	0.12	0.47
Greece	0.10	0.73
Italy	0.34	0.44
Japan	0.23	0.53
Korea	0.34	0.35
Malta	0.01	0.02
Netherlands	0.83	0.83
Portugal	0.13	0.22
Spain	0.92	0.92
Sweden	0.21	0.44
United Kingdom	0.39	0.51
United States	0.36	0.72

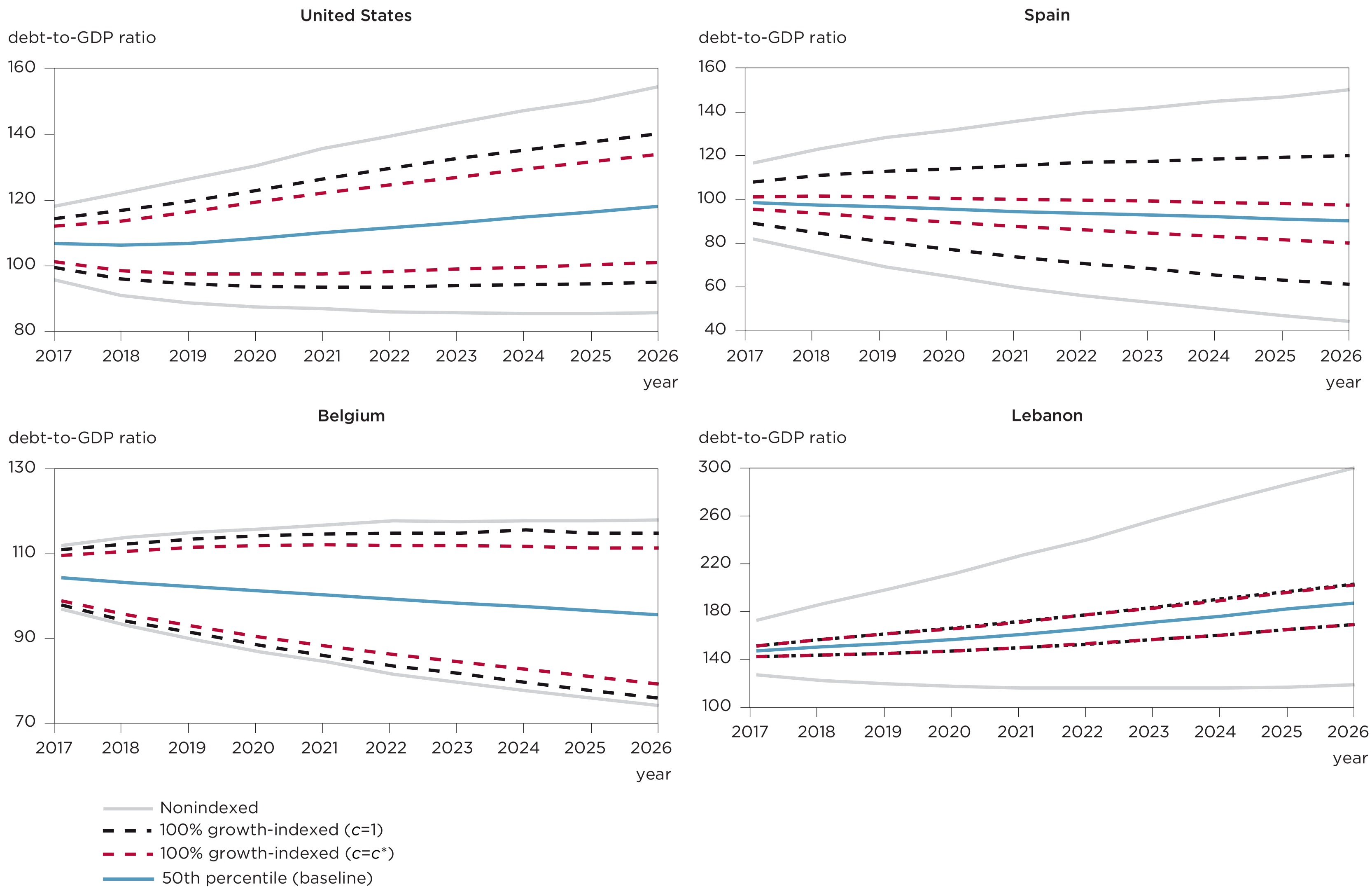
Developing economies

	Growth rate (g)
Argentina	0.04
Brazil	0.68
Cameroon	0.01
Chile	0.40
Colombia	0.00
Costa Rica	0.76
Egypt	0.44
India	0.10
Indonesia	0.06
Israel	0.21
Lebanon	0.02
Mexico	0.34
Peru	0.37
South Africa	0.58
Turkey	0.42

Note: Shaded cells indicate R^2 statistics above 0.60.

Sources: International Monetary Fund *World Economic Outlook* database, April 2017; and author's calculations.

Figure 3 Debt-to-GDP ratio forecasts for growth-indexed debt with different indexation coefficients



c = indexation coefficient, or the elasticity of the return to the growth rate; c^* = optimal indexation coefficient

Note: The lines represent the 1st and 99th percentiles of the debt-to-GDP distribution where the debt is nonindexed (grey), all the debt is indexed with an indexation coefficient $c = 1$ (black), and all the debt is indexed with an indexation coefficient $c = c^*$ (red). By construction the baseline (median) is the same for all cases (blue).

Source: Author's calculations.

Table 5 Percentile of the nonindexed distribution corresponding to the 99th percentile of the indexed distribution for growth-indexed bonds with optimal coefficient

	c coefficient value		
Country	**1**	***c****	**Difference**
Spain	89	62	–27
Netherlands	95	74	–21
Costa Rica	90	72	–18
Brazil	81	68	–13
France	91	81	–10
South Africa	94	85	–9
Turkey	94	87	–7
United States	92	85	–7
Italy	89	83	–6
Mexico	84	79	–5
Peru	94	89	–5
United Kingdom	96	91	–5
Chile	95	91	–4
Indonesia	90	86	–4
Australia	95	92	–3
Canada	94	91	–3
Japan	78	75	–3
Portugal	85	82	–3
Belgium	97	95	–2
Egypt	74	72	–2
Austria	92	91	–1
Germany	91	90	–1
Greece	75	74	–1
India	91	90	–1
Israel	92	91	–1
Korea	98	97	–1
Lebanon	65	64	–1
Sweden	97	96	–1
Argentina	80	80	0
Cameroon	98	98	0
Colombia	91	91	0
Malta	92	92	0

c : indexation coefficient, i.e. the elasticity of the return to the growth rate; *c** : optimal indexation coefficient

Note: The numbers represent the percentile of the nonindexed distribution corresponding to the 99th percentile of the indexed distribution after 10 years, assuming iid shocks and 100 percent indexed debt.

Source: Author's calculations.

Table 6 Optimal indexation coefficients to the nominal growth rate and the output gap, when all debt is indexed

	Optimal indexation coefficients	
Country	**Growth rate (*a**)**	**Output gap (*b**)**
Austria	1.04	0.44
Belgium	1.78	0.19
France	1.55	0.28
Germany	0.80	1.20
Greece	1.17	–0.27
Italy	1.41	–0.24
Japan	1.05	0.29
Malta	1.08	–0.15
Netherlands	2.27	0.04
Portugal	1.31	–0.27
Spain	2.04	–0.07
Sweden	1.10	1.23
United Kingdom	1.67	0.96
United States	1.06	1.15

Note: In the *a** and *b** formulas, dt–1 is fixed to the debt-to-GDP ratio in 2016 in order to have a fixed (nontime varying) coefficient. Although this tends to decrease the efficiency of the optimal coefficient as the debt ratio deviates from its initial level, this effect is limited.

Source: Author's calculations.

Table 7 Correlation between the nominal growth rate (g) and the output gap (z)

Country	Correlation
Austria	0.61
Belgium	0.70
France	0.70
Germany	0.66
Greece	0.33
Italy	0.47
Japan	0.55
Malta	0.21
Netherlands	0.65
Portugal	0.62
Spain	0.69
Sweden	0.65
United Kingdom	0.57
United States	0.73

Sources: International Monetary Fund *World Economic Outlook* database, April 2017; and author's calculations.

Table 8 Percentile of the nonindexed distribution corresponding to the 99th percentile of the indexed distribution for growth-indexed and growth-gap-indexed bonds with optimal coefficients

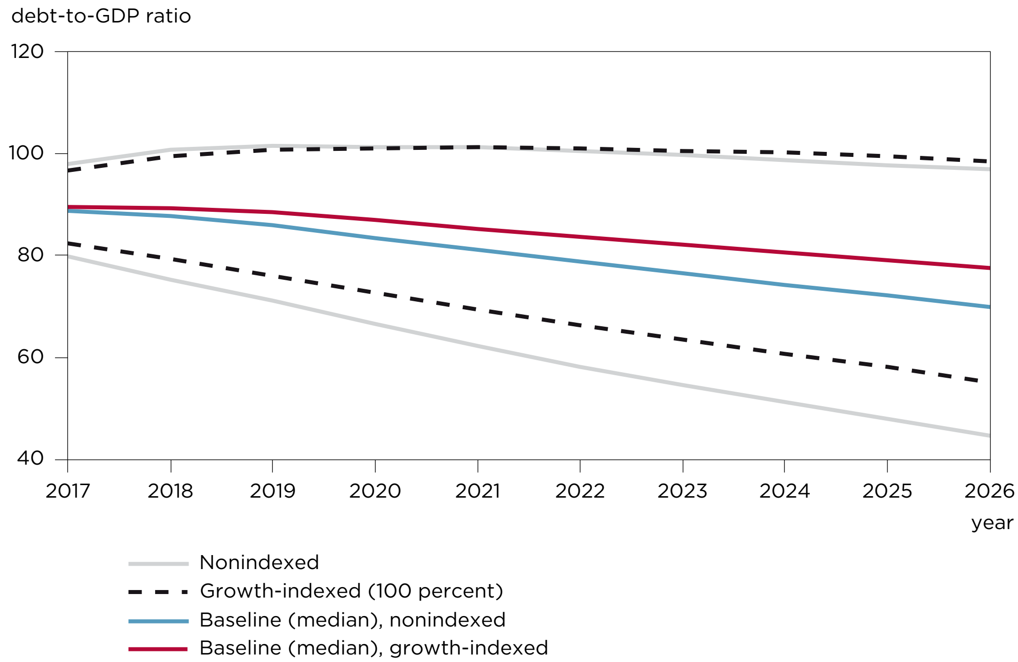
	Optimal coefficients		
Country	***c****	***a*****, *b*****	**Difference**
Greece	74	62	–12
United States	88	79	–9
Germany	89	84	–5
Japan	73	68	–5
Italy	85	82	–3
Sweden	96	93	–3
Austria	91	89	–2
United Kingdom	93	91	–2
Belgium	95	94	–1
France	83	82	–1
Portugal	83	82	–1
Malta	91	91	0
Netherlands	75	75	0
Spain	61	61	0

c*: optimal indexation coefficient to the growth rate; *a**, *b**: optimal indexation coefficients to the growth rate and the output gap

Note: The numbers represent the percentile of the nonindexed distribution corresponding to the 99th percentile of the indexed distribution after 10 years, assuming iid shocks, 100 percent indexed debt and formulas with optimal coefficients.

Source: Author's calculations.

Figure 4 Impact of a 100-basis-point premium on the distribution of debt ratio forecasts for the United Kingdom



Note: The lines represent the 1st and 99th percentiles of the debt-to-GDP distribution for debt that is nonindexed debt (grey), all growth-indexed (black), [The blue line represents the baseline without indexed debt. The red line represents the baseline with indexation, assuming indexed bonds pay a 100 basis points premium].

Source: Author's calculations.

Table 9 Maximum premium on growth-indexed bonds that would equalize the upper tail of the distribution in the nonindexed and indexed cases

Country	99th percentile, 10-year horizon	95th percentile, 20-year horizon
Lebanon	4.1	2.2
Argentina	5.4	2.2
Brazil	2.8	1.6
Greece	3.0	1.6
Egypt	2.6	1.5
Mexico	2.3	1.3
Turkey	2.2	1.2
Spain	2.3	1.2
Colombia	1.7	1.0
Indonesia	2.3	1.0
Portugal	1.6	0.9
Malta	1.6	0.9
Peru	1.8	0.8
Chile	1.9	0.8
Costa Rica	1.7	0.8
Germany	1.3	0.7
South Africa	1.2	0.7
Australia	1.4	0.7
Japan	1.2	0.6
Netherlands	1.2	0.6
Italy	0.9	0.5
India	1.1	0.5
United States	1.0	0.5
Canada	1.0	0.5
Israel	1.0	0.4
France	0.8	0.4
United Kingdom	0.8	0.3
Austria	0.7	0.3
Sweden	0.5	0.3
Belgium	0.3	0.1
Cameroon	0.2	0.1
Korea	0.0	–0.9

t = forecast horizon

Note: All debt is indexced (X = 1). Maximum premium = (nonindexed debt ratio_t / debt ratio_t0)^(1/t) – (indexed debt ratio_t / debt ratio_t0)^(1/t)).

Source: Author's calculations.

Appendix 1 Formula Derivations

Model Assumptions

Consider the case of an indebted and risk-averse sovereign that issues one-period plain vanilla debt. The debt is rolled over every period and is denominated in local currency. The sovereign prefers to avoid costly default or sharp fiscal adjustment and thus prefers decreasing to increasing debt ratio levels. Moreover, the sovereign is risk averse and thus prefers low variance in the debt-to-GDP ratio, i.e. low uncertainty regarding the future value of the debt-to-GDP ratio. Thus the utility function of the sovereign is decreasing in the debt-to-GDP ratio and concave.

The impact of the issuance of different types of one-period and multiple-period growth-indexed bonds on the variance of the debt-to-GDP ratio is derived. Bondholder investors are assumed to be risk neutral. For simplicity, it is assumed without loss of generality that the debt-to-GDP ratio is expected to remain constant. Moreover, the variance-covariance matrix—consisting of the variances and covariances of the nominal growth rate, g_t, the nominal interest rate on outstanding debt, r_t, and the primary balance as a share of GDP, pb_t— is assumed to be given.

Debt Dynamics with Plain-vanilla Bonds

The central government debt level at time t, denoted by D_t, is equal to the debt level at time $t-1$ plus the corresponding nominal interest paid on the outstanding debt $r_t D_{t-1}$ minus the primary balance surplus PB_t:

$$D_t = (1 + r_t)D_{t-1} - PB_t \tag{27}$$

Normalizing by the GDP level at time t obtains:

$$\frac{D_t}{GDP_t} = \frac{(1 + r_t)D_{t-1}}{GDP_t} - \frac{PB_t}{GDP_t} \tag{28}$$

$$\frac{D_t}{GDP_t} = \frac{(1 + r_t)D_{t-1}}{(1 + g_t)GDP_{t-1}} - \frac{PB_t}{GDP_t} \tag{29}$$

The equation driving the debt-to-GDP ratio dynamics is:

$$d_t = \frac{1 + r_t}{1 + g_t} d_{t-1} - pb_t \tag{30}$$

If the nominal interest rate r_t and the nominal growth rate g_t are relatively small numbers, this is approximately equivalent to:

$$d_t = (1 + r_t - g_t)d_{t-1} - pb_t \tag{31}$$

$$\Delta d_t = (r_t - g_t)d_{t-1} - pb_t \tag{32}$$

It is assumed without loss of generality that the debt-to-GDP ratio is expected to remain constant and thus that the expected change in the public debt ratio is equal to zero. The variance in the change in the public debt-to-GDP ratio can be computed by using the multivariate delta method, or variance and covariance properties, and is equal to:

$$var(\Delta d_t) = var((r - g)d_{t-1} - pb) \tag{33}$$

$$var(\Delta d_t) = var(pb) + d_{t-1}^2 var(r - g) - 2d_{t-1}cov(pb, r - g) \tag{34}$$

Debt Dynamics with Simple GIBs

Consider that a share X of the stock of debt is composed of simple growth-indexed bonds (GIBs). The indexation formula is specified such that the risk-neutral bond-holder is indifferent between investing in a one-period fixed interest bond or a one-period GIB at time $t-1$:

$$rind_t = g_t + k_t \tag{35}$$

Where k_t (known at time $t-1$) is such that the expected return on a GIB at time t is equal to the return on a fixed interest rate bond at time t (which is also known at time $t-1$). Note that k_t is different from a default, risk, liquidity, or novelty premium. Thus:

$$r_t = \overline{rind}_t = \overline{g}_t + k_t \tag{36}$$

Where the operator $\overline{\bullet}_t$ denotes the expectation made at time $t-1$ of the value of a variable at time t. Thus k_t, which equalizes the expected yield on both one-period bonds, is equal to:

$$k_t = r_t - \overline{g}_t \tag{37}$$

The debt dynamics equation becomes:

$$\Delta d_t = [X(rind_t - g_t) + (1-X)(r_t - g_t)]d_{t-1} - pb_t \tag{38}$$

$$\Delta d_t = [(Xk_t) + (1-X)(r_t - g_t)]d_{t-1} - pb_t \tag{39}$$

$$\Delta d_t = [(r_t - g_t) + X(g_t - \overline{g}_t)]d_{t-1} - pb_t \tag{40}$$

As $r_t = \overline{rind}_t$, the expected change in the public debt ratio is equal to zero as with plain-vanilla bonds. The variance in the change in public debt-to-GDP ratio is equal to:

$$\begin{aligned} var(\Delta d_t) = {} & var(pb) + d_{t-1}^2 var(r-g) - 2d_{t-1}cov(pb, r-g) + d_{t-1}^2 X^2 var(g) \\ & + 2d_{t-1}^2 X cov(r,g) - 2d_{t-1}^2 X var(g) - 2d_{t-1} X cov(g, pb) \end{aligned} \tag{41}$$

Note that if $X = 1$, i.e if all the stock of debt is composed of simple GIBs, the debt dynamics equation becomes:

$$\Delta d_t = (k_t)d_{t-1} - pb_t \tag{42}$$

$$\Delta d_t = (r_t - \overline{g}_t)d_{t-1} - pb_t \tag{43}$$

Moreover the variance reduces to:

$$var(\Delta d_t) = var(pb) + d_{t-1}^2 var(r) - 2d_{t-1} cov(pb, r) \tag{44}$$

In this case, the debt-to-GDP ratio does not depend on the realized nominal growth rate. Thus the sovereign is fully hedged against shocks to the growth rate.

Finally, if it is assumed that the sovereign is able to issue long-term GIBs (with a multiple-period maturity) and does not have to issue new debt until maturity, then $k_t = k$ is a constant over the considered period. To be more explicit, k would be a constant such that the risk-neutral investor is indifferent between holding a GIB until maturity and rolling over one-period nonindexed bonds over the same period.[16] In this case, the variance of the debt-to-GDP ratio is:

$$var(\Delta d_t) = var(pb) + (1 - X)^2 d_{t-1}^2 var(r - g) - 2(1 - X)d_{t-1} cov(pb, r - g) \tag{45}$$

Note that if $X = 1$, the sovereign is fully hedged against shocks to the growth rate and the interest rate. The variance of the debt-to-GDP ratio collapses to the variance of the primary balance as a share of GDP:

$$var(\Delta d_t) = var(pb) \tag{46}$$

Second-order Stochastic Dominance

As k_t (respectively k for the long-term indexed bond) is defined such that the investor is indifferent between investing in the fixed interest rate bond and the

[16] This k is predetermined at issuance and is not revised afterwards, thus it does not contribute to the variance of the debt-to-GDP ratio.

growth-indexed bond (GIB), the expected value of the debt-to-GDP ratio in the next period (respectively at maturity) is the same in both cases. Thus a sufficient condition for the issuance of simple growth-indexed bonds to be preferred by the risk-averse sovereign by second-order stochastic dominance is that the variance of the change in the debt-to-GDP ratio is lower than in the nonindexed case.

Using equations (34) and (41), the issuance of a share X of simple growth-indexed bonds is preferred to the nonindexed case by the sovereign by second-order stochastic dominance if:

$$2d_{t-1}X cov(g, pb) - 2d_{t-1}^2 X cov(r, g) + (2d_{t-1}^2 X - d_{t-1}^2 X^2) var(g) > 0 \tag{47}$$

$$X < 2 + \frac{2cov(pb, g)}{d_{t-1}var(g)} - \frac{2cov(r, g)}{var(g)} \tag{48}$$

Note that the share X can be as high as 1 if the following condition holds:

$$1 + \frac{2cov(pb, g)}{d_{t-1}var(g)} - \frac{2cov(r, g)}{var(g)} > 0 \tag{49}$$

Consider now the case of long-term GIBs. Using equations (34) and (45), the issuance of a share X of simple long-term growth-indexed bonds is preferred to the issuance of short-term nonindexed bonds by the sovereign by second-order stochastic dominance if:

$$[1 - (1 - X)^2] d_{t-1}^2 var(r - g) - 2[1 - (1 - X)] d_{t-1} cov(pb, r - g) > 0 \tag{50}$$

$$X < 2 - \frac{2cov(pb, r - g)}{d_{t-1}var(r - g)} \tag{51}$$

Note that the share X can be as high as 1 if the following condition holds:

$$1 - \frac{2cov(pb, r - g)}{d_{t-1}var(r - g)} > 0 \tag{52}$$

The optimal share $X^\star$ that achieves the lowest variance in the debt-to-GDP ratio is given by the first-order conditions of the minimization of the variance of changes in the debt ratio with respect to X. The optimal share is obtained by taking the first derivative of the left-hand side of inequality (47) with respect to X:

$$0 = d_{t-1}^2(2 - 2X)var(g) + 2d_{t-1}cov(pb, g) - 2d_{t-1}^2 cov(r, g) \tag{53}$$

$$X^\star = 1 + \frac{cov(pb, g)}{d_{t-1}var(g)} - \frac{cov(r, g)}{var(g)} \tag{54}$$

In the case of long-term GIBs, the optimal share $X^\star$ is given by the first-order conditions of the minimization of the variance of changes in the debt ratio with respect to X. The optimal share is obtained by taking the first derivative of the left-hand side of inequality (50) with respect to X:

$$X^\star = 1 - \frac{cov(pb, r - g)}{d_{t-1}var(r - g)} \tag{55}$$

Debt Dynamics with Alternative Indexation Formulas

This section explores alternative indexation formulas, the design of which is more complex than the simple growth-indexed bond presented above.

A. The Fully Contingent Formula

This is the indexation formula such that:

$$var(\Delta d_t) = 0 \tag{56}$$

$$\Delta d_t = a \tag{57}$$

Where a is a constant. If it is assumed without loss of generality that the debt-to-GDP ratio is expected to remain constant, then:

$$\Delta d_t = 0 \tag{58}$$

Using equations (32) and (58) obtains:

$$rind_t = \frac{pb_t}{d_{t-1}} + g_t \tag{59}$$

This indexation formula achieves the highest reduction in the variance of the debt ratio. In fact, the variance is equal to zero. This formula provides an important insight: The optimal formula indexes the interest rate not only to the growth rate but also to the primary balance and the lagged debt-to-GDP ratio.

B. Growth-indexed Bonds

In the case of a growth-indexed bond, the equation is:

$$rind_t = cg_t + k_t \tag{60}$$

Note that k_t is now equal to:

$$k_t = r_t - c\overline{g}_t \tag{61}$$

Where c is the indexation coefficient, or the elasticity of the return to the growth rate. k_t is defined such that the investor is indifferent between investing in the one-period fixed interest rate bond and the one-period growth-indexed bond. In the simplest case analyzed above $c = 1$, but in theory this coefficient could take any positive value.

The debt dynamics equation becomes:

$$\Delta d_t = [(r_t - g_t) + cX(g_t - \overline{g}_t)]d_{t-1} - pb_t \tag{62}$$

Again, the expected change in the public debt ratio is equal to zero. The variance in the change in public debt-to-GDP ratio is equal to:

$$var(\Delta d_t) = var(pb) + d_{t-1}^2 var(r-g) - 2d_{t-1}cov(pb, r-g) + d_{t-1}^2 (cX)^2 var(g) \\ +2d_{t-1}^2 cX cov(r,g) - 2d_{t-1}^2 cX var(g) - 2d_{t-1} cX cov(g, pb) \quad (63)$$

The optimal indexation coefficient $c^\star$ is given by the first-order conditions of the minimization of the variance of changes in the debt ratio with respect to c. Using equations (34) and (63), the issuance of a share X of growth-indexed bonds is preferred to the nonindexed case by the sovereign by second-order stochastic dominance if:

$$2d_{t-1}cX cov(g, pb) - 2d_{t-1}^2 cX cov(r,g) + (2d_{t-1}^2 cX - d_{t-1}^2 (cX)^2) var(g) > 0 \quad (64)$$

Taking the derivative of (64) with respect to c obtains:

$$0 = d_{t-1}^2 (2X - 2cX^2) var(g) + 2X d_{t-1} cov(pb, g) - 2X d_{t-1}^2 cov(r,g) \quad (65)$$

After dividing by $2Xd_{t-1}^2$ and rearranging, the optimal coefficient is:

$$c^\star = \frac{1}{X} + \frac{cov(g, pb)}{X d_{t-1} var(g)} - \frac{cov(r,g)}{X var(g)} \quad (66)$$

As previously discussed, if it is further assumed that the sovereign is able to issue long-term GIBs (with a multiple-period maturity) and does not have to issue new debt until maturity, then the debt dynamics equation can be written:

$$\Delta d_t = [Xk + (1-X)r_t - (1-cX)g_t]d_{t-1} - pb_t \quad (67)$$

Again, the expected change in the public debt ratio is equal to zero. The variance in the change in public debt-to-GDP ratio is equal to:

$$var(\Delta d_t) = var(pb) + (1-X)^2 d_{t-1}^2 var(r) - (1-Xc)^2 d_{t-1}^2 var(g) \\ -2d_{t-1}^2 (1-X)(1-Xc) cov(r,g) + 2d_{t-1}(1-Xc) cov(g, pb) - 2d_{t-1}(1-X) cov(pb, r) \quad (68)$$

The optimal indexation coefficient $c^\star$ is given by the first-order conditions of the minimization of the variance of changes in the debt ratio with respect to c. By taking the derivative of (68) with respect to c, and after rearranging, the equation obtained is:

$$c^{\star} = \frac{1}{X} + \frac{cov(g, pb)}{X d_{t-1} var(g)} - \frac{(1-X)cov(r, g)}{X var(g)} \tag{69}$$

Note that if $X = 1$, the optimal indexation coefficient $c^{\star}$ simplifies to:

$$c^{\star} = 1 + \frac{cov(g, pb)}{d_{t-1} var(g)} \tag{70}$$

Moreover, the variance using this optimal coefficient $c^{\star}$ is:

$$var(\Delta d_t) = var(pb) + d_{t-1}^2 (\frac{cov(g, pb)}{d_{t-1} var(g)})^2 var(g) - 2d_{t-1}(\frac{cov(g, pb)}{d_{t-1} var(g)})cov(pb, g) \tag{71}$$

$$var(\Delta d_t) = var(pb) + \frac{(cov(g, pb))^2}{var(g)} - \frac{2(cov(g, pb))^2}{var(g)} \tag{72}$$

$$var(\Delta d_t) = var(pb) - \frac{(cov(g, pb))^2}{var(g)} \tag{73}$$

$$var(\Delta d_t) = var(pb) - var(pb)(\rho_{g,pb})^2 \tag{74}$$

$$var(\Delta d_t) = var(pb)(1 - (\rho_{g,pb})^2) \tag{75}$$

Where $\rho_{g,pb}$ is the correlation between the nominal growth rate and the primary balance as a share of GDP.

C. Growth-gap Indexed Bonds

In the case of a long-term growth-gap indexed bond, the equation is:

$$rind_t = a g_t + b z_t + k \tag{76}$$

Where z_t is the output gap at time t, a is the indexation coefficient to the growth rate, and b the indexation coefficient to the output gap. The output gap is defined as the difference, in percentage points, between current and potential output. A positive value is defined as current output above potential output. k

is a constant such that the risk-neutral investor is indifferent between holding this long-term growth-gap indexed bond until maturity and rolling over one-period nonindexed bonds over the same period.

The debt dynamics equation becomes:

$$\Delta d_t = [X(rind_t - g_t) + (1 - X)(r_t - g_t)]d_{t-1} - pb_t \tag{77}$$

$$\Delta d_t = [X((a - 1)g_t + bz_t + k) + (1 - X)(r_t - g_t)]d_{t-1} - pb_t \tag{78}$$

And thus the variance in the change in public debt-to-GDP ratio is equal to:

$$\begin{aligned} var(\Delta d_t) = var(pb) + (Xd_{t-1})^2(b^2var(z) + (a-1)^2var(g) + 2(a-1)bcov(z,g)) \\ +((1-X)d_{t-1})^2var(r-g) \\ +2X(1-X)d_{t-1}^2[(a-1)(cov(g,r) - (var(g)) + b(cov(z,r) - (cov(z,g))] \\ -2Xd_{t-1}((a-1)cov(pb,g) + bcov(pb,z)) \\ -2(1-X)d_{t-1}(cov(pb,r) - cov(pb,g)) \end{aligned} \tag{79}$$

Note that if $X = 1$, the variance simplifies to:

$$\begin{aligned} var(\Delta d_t) = var(pb) + d_{t-1}^2(b^2var(z) + (a-1)^2var(g) + 2(a-1)bcov(z,g)) \\ -2d_{t-1}((a-1)cov(pb,g) + bcov(pb,z)) \end{aligned} \tag{80}$$

Moreover, the optimal indexation coefficients $a^\star$ and $b^\star$ satisfy the following conditions:

$$a^\star = 1 + \frac{cov(pb,g)var(z) - cov(pb,z)cov(z,g)}{d_{t-1}(var(g)var(z) - cov(z,g)^2)} \tag{81}$$

$$b^\star = \frac{cov(pb,z)var(g) - cov(pb,g)cov(z,g)}{d_{t-1}(var(g)var(z) - cov(z,g)^2)} \tag{82}$$

More formally, if the correlation between the growth rate and the output gap is equal to zero, then the variance using optimal coefficients $a^\star$ and $b^\star$ is:

$$var(\Delta d_t) = var(pb)(1 - (\rho_{g,pb})^2 - (\rho_{z,pb})^2) \tag{83}$$

Where $\rho_{g,pb}$ is the correlation between the nominal growth rate and the primary balance as a share of GDP and $\rho_{z,pb}$ is the correlation between the output gap and the primary balance as a share of GDP.

Note that $b^\star = 0$ if $cov(pb, z)var(g) = cov(pb, g)cov(z, g)$, i.e. if:

$$\rho_{pb,z} = \rho_{pb,g}\rho_{g,z} \tag{84}$$

Similarly, note that $a^\star = 1$ if $cov(pb, g)var(z) = cov(pb, z)cov(z, g)$, i.e. if:

$$\rho_{pb,g} = \rho_{pb,z}\rho_{g,z} \tag{85}$$

By combining equations (54) and (53), $a^\star = 1$ and $b^\star = 0$ are obtained if: $|\rho_{g,z}| = 1$ and $\rho_{pb,g} = \rho_{pb,z} = 0$.

D. Identically and Independently Distributed Shocks

This section describes the technical assumptions underlying the Monte Carlo simulations.

Throughout the simulation window, 10 or 20 years starting in 2017, the expected values of the nominal effective interest rate, the nominal growth rate, and the ratio of the primary balance to GDP, are taken to be equal to the International Monetary Fund's April 2017 *World Economic Outlook* forecasts up to 2022, the last year available going forward, and are extrapolated at the same values from then on.

The nominal effective interest rate is constructed as the difference between the primary balance and the overall fiscal balance over the lagged debt level. This is only an approximation as the difference corresponds to net interest paid rather than gross interest paid.

The distribution of shocks for the nominal effective interest rate, the nominal growth rate, and the ratio of the primary balance to GDP is assumed to be a multivariate normal distribution, with a covariance matrix given by the empirical covariance matrix estimated over 1996–2016. The shocks are assumed to be independently and identically distributed over time, that is, shocks occurring this year have no implications for the distribution of shocks next year.

The interest rate in all indexed contacts includes a constant k, where the expected return on a long-term indexed bond is equal to the expected return of rolling over short-term nominal bonds over the same period. This implies that, absent a potential premium on indexed debt contracts, the expected (median) debt-to-GDP path is the same under indexation and nonindexation.

The projected paths for the debt-to-GDP ratio are then generated through 10,000 random draws from the multivariate distribution. This methodology is used for growth-indexed bonds, with an indexation equal to 1 or to the optimal coefficient denoted by $c^{\star}$. The sample includes the following 32 countries: Argentina, Australia, Austria, Belgium, Brazil, Cameroon, Canada, Chile, Colombia, Costa Rica, Egypt, France, Germany, Greece, India, Indonesia, Israel, Italy, Japan, Korea, Lebanon, Malta, Mexico, the Netherlands, Peru, Portugal, South Africa, Spain, Sweden, Turkey, the United Kingdom, and the United States.

E. Vector Autoregression with One Lag VAR(1)

A similar analysis for the same group of countries is replicated under the assumption that shocks have intertemporal effects, i.e. that shocks occurring this year have implications for the distribution of the estimated variables in the future. The results are reported in appendix 2. A 3-variable vector autoregression with one lag VAR(1) is estimated for each country, using annual data from 1996 to 2016, for the nominal effective interest rate, the nominal growth rate, and the primary balance as a share of GDP.

For each period, estimates from the VAR are adjusted in order to equalize the expected value for the nominal effective interest rate, the nominal growth rate, and the primary balance as a share of GDP to the IMF forecast. The regressions parameters and the covariance matrix of the errors are unchanged, but a time-varying constant is imposed for each variable.

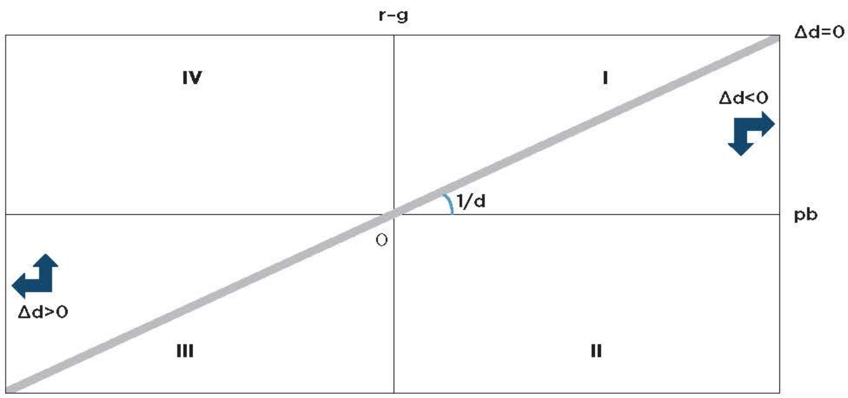
A 4-variable vector autoregression with one lag is estimated for a subsample of 14 countries, using annual data from 1996 to 2016, for the nominal effective interest rate, the nominal growth rate, the primary balance as a share of GDP, and the output gap as a share of potential GDP. The subsample includes the following countries: Austria, Belgium, France, Germany, Greece, Italy, Japan, Malta, the Netherlands, Portugal, Spain, Sweden, the United Kingdom, and the United States. In this specification, the nominal effective interest rate is constructed as the ratio of gross interest payments over the lagged debt level. This variable is computed by Eurostat and extracted from the AMECO database. No forecasts are available after 2018 for the interest rate from AMECO. Thus a regression of the interest rate from AMECO is made on the approximation previously obtained by using IMF data, and this linear regression and IMF forecasts are used to compute forecasts for the AMECO interest rate. The R^2 statistics from a regression of the AMECO interest rate on the IMF approximation is higher than 0.85 for all 14 countries in the subsample. Whenever no forecast for the output gap is available for one of the 14 countries, the IMF forecast is extrapolated by assuming that the output gap follows an autoregressive process with one lag and a coefficient equal to 0.9.

The source for all other variables is the IMF WEO April 2017 database. This methodology is used for growth-indexed bonds and the two alternative formulas described in the text.

For some countries, the VAR(1) is not stationary, as it contains a unit root. The concerned countries are Argentina, Brazil, Belgium, Germany, and Sweden for the 3-variable specification. The two concerned countries are Austria and the Netherlands for the 4-variable specification. As a consequence, the parameter estimates are not well-behaved in the sense that variables follow explosive paths, implying diverging debt-to-GDP ratios. This is explained by the short estimation window, limited numbers of points, and the nonstandard behavior of macroeconomic variables in some countries over the recent period. For example, while the primary balance appears to be nonstationary in Brazil, it is very unlikely that the primary deficit as a share of GDP will continue to increase over time. In most cases, however, the gains in terms of reduction in the upper-tail of the debt distribution are similar under both specifications.

A graphical representation

Figure A1.1 Debt variation: Nonindexed debt

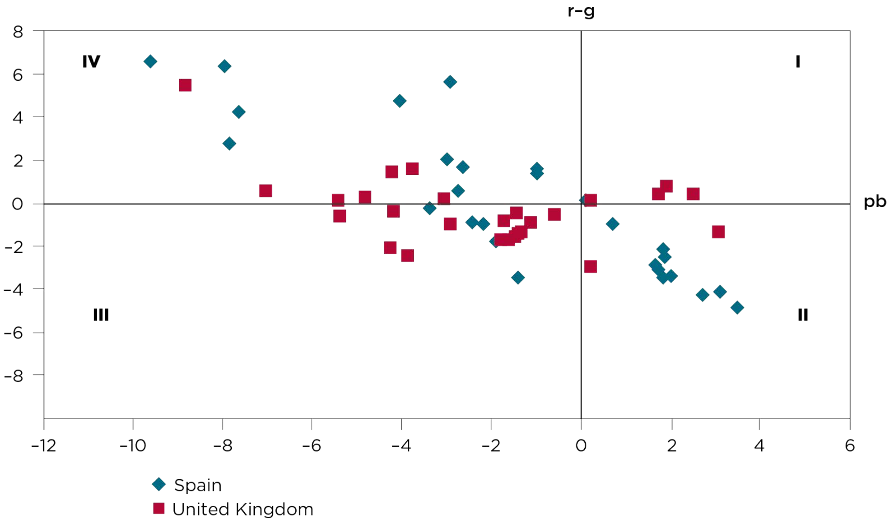


r = real implicit interest rate; g = GDP growth rate; d = debt; pb = primary balance
Source: Author's calculations.

The debt dynamics equation can be represented conceptually by using a scatterplot. In figure A1.1, the grey line $\Delta d = 0$ gives the combinations of the real implicit interest rate minus the real GDP growth rate and the primary balance where the debt variation is equal to zero for a given debt level. This line has a positive slope equal to the inverse of the debt-to-GDP ratio. For any point below this line, the debt decreases, while it increases above this line.

The further a point is away from this line, the higher the change in the debt ratio. In particular, points in quadrant II (lower right) would imply high reductions in the debt ratio, while points in quadrant IV (upper left) would imply high increases in the debt ratio. A wide range of values for the interest-growth differential and the primary balance, and a negative correlation between the interest-growth differential and the primary balance, would then imply a high variance in the variability of the debt ratio. Conversely, a narrow range of values for the interest-growth differential and for the primary balance, and a positive correlation between the interest-growth differential and the primary balance, would then imply a low variance in the variability of the debt ratio. Scatterplots covering the period from 1990 to 2016 for Spain and the United Kingdom show that the potential benefits from indexation vary from one country to another. Spain, which has many points in quadrants II and IV, was more likely than the United Kingdom to benefit from debt indexation over that period (figure A1.2).

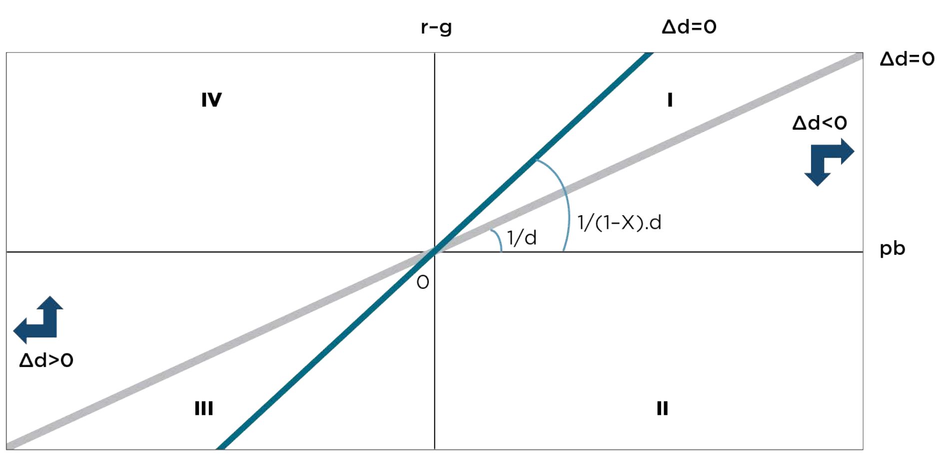
Figure A1.2 Interest growth versus primary balance (1990–2016)



r = real implicit interest rate; g = GDP growth rate; pb = primary balance
Source: International Monetary Fund *World Economic Outlook* database.

On the graphical representation, simple growth-indexed bonds would reduce the distance from every point to the horizontal axis by X percent. Put another way, and assuming for simplicity that k is equal to zero, this is equivalent to shifting the grey line to the blue one (figure A1.3). The intuition is simple: For a given positive interest-growth differential (r-g), i.e. if growth is lower than the interest rate, the primary balance needed to leave the debt level unchanged is lower than under nonindexation. Conversely, for a given negative r-g, i.e. if growth is higher than the interest rate, the primary balance needed to leave the debt level unchanged is higher than under nonindexation. A positive k would move the blue line to the right, thus reducing the gains for a positive r-g and increasing the fiscal cost for a negative r-g. A positive premium would have the same impact.

Figure A1.3 Debt variation: Nonindexed debt (grey) versus X percent growth-indexed debt (blue)

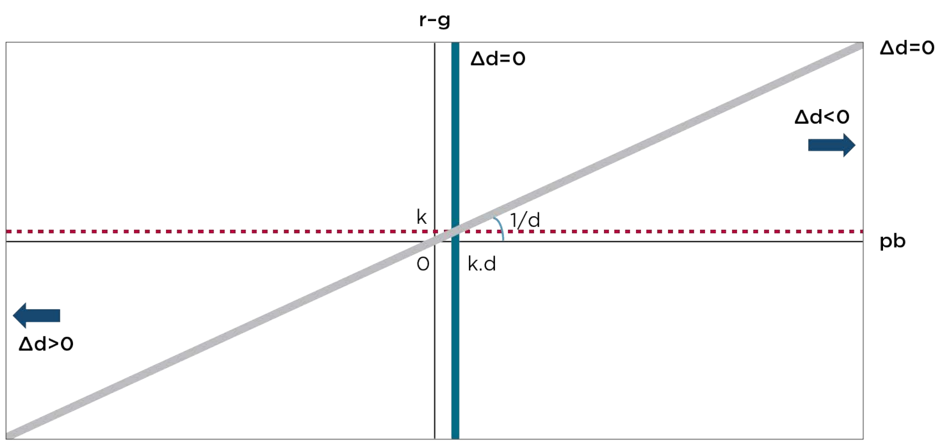


r = real implicit interest rate; g = GDP growth rate; d = debt; pb = primary balance
Source: Author's calculations.

Take a simple example to illustrate this point. Assume that the debt-to-GDP ratio is equal to 100 percent and that 50 percent of the stock of debt is composed of growth-indexed bonds. Also, assume for simplicity that k is equal to zero. If the difference between the interest rate on nominal bonds and the growth rate is -2 percent, then the primary deficit that would keep the debt-to-GDP ratio unchanged would be 1 percent with indexation, compared to 2 percent without indexation. Conversely, if the difference between the interest rate on nominal bonds and the growth rate is 6 percent, the primary surplus needed to keep the debt-to-GDP ratio unchanged would be 3 percent with indexation, compared to 6 percent without indexation.

Another argument can be made here. Assuming unchanged parameters, the debt-to-GDP ratio would have to be as high as 200 percent in the indexed scenario, compared to 100 percent in the nonindexed scenario, to require a primary surplus of 6 percent to stabilize the debt-to-GDP ratio. In that sense, growth-indexed bonds increase fiscal space, defined as the difference between the current debt level and a theoretical debt limit, i.e. the threshold above which a country can meet its debt service obligations only by an extraordinary fiscal effort (Pienkowski 2017). If all the stock of debt is composed of growth-indexed bonds (figure A1.4), the scatterplot would simplify to the red horizontal line crossing the vertical axis at k, as in this case r-g is always equal to this constant. The grey line would de facto shift to a vertical line as the interest-growth differential is no longer stochastic but fixed to k. Changes in the debt ratio would be determined entirely by the primary balance, and the variance of unexpected changes in the debt ratio would then be equal to the variance of the primary balance.

Figure A1.4 Debt variation: 100 percent growth-indexed debt



r = real implicit interest rate; g = GDP growth rate; d = debt; pb = primary balance; k = [constant]

Source: Author's calculations.

APPENDIX 2 TABLES

Table A2.1 Percentile of the nonindexed distribution corresponding to the 99th percentile of the indexed distribution — Growth–indexed bonds with different shares of indexed debt — VAR(1) against iid shocks specification

VAR(1) specification			Difference between the specifications[a]		
percent indexed debt:	100	20	percent indexed debt:	100	20
Lebanon	58	97	Lebanon	–7	0
Greece	72	97	Australia	–5	0
Egypt	72	97	Chile	–4	0
Japan	80	97	Italy	–3	–1
Brazil	82	97	Greece	–3	0
Italy	86	97	Egypt	–2	0
Portugal	87	98	Netherlands	–1	0
Spain	88	98	Malta	–1	0
Argentina	88	96	Spain	–1	0
Australia	90	98	United Kingdom	0	0
Malta	91	98	Belgium	0	0
Chile	91	98	Austria	1	0
India	92	98	Canada	1	0
Indonesia	92	98	Brazil	1	0
Austria	93	98	India	1	0
Costa Rica	93	98	Korea	1	1
United States	94	98	Cameroon	1	1
Netherlands	94	98	United States	2	0
Colombia	94	98	Sweden	2	0
Israel	94	98	Japan	2	0
France	95	98	Portugal	2	1
Canada	95	98	Turkey	2	0
Mexico	95	98	South Africa	2	0
United Kingdom	96	98	Israel	2	0
Germany	96	98	Indonesia	2	0
Turkey	96	98	Colombia	3	0
South Africa	96	98	Costa Rica	3	0
Belgium	97	98	Peru	3	0
Peru	97	98	France	4	0
Sweden	99	98	Germany	5	0
Korea	99	99	Argentina	8	–1
Cameroon	99	99	Mexico	11	1

VAR = vector autoregression

a. A negative value means that the reduction in the variance of the indexed debt distribution is more important under the VAR specification. Conversely, a positive value means that the reduction in the variance of the indexed debt distribution is more important under the iid shocks specification.

Source: Author's calculations.

Table A2.2 Percentile of the nonindexed distribution corresponding to the 99th percentile of the indexed distribution — growth-indexed bonds with optimal indexation coefficient — VAR(1) against iid shocks specification

VAR(1) specification – 100 percent indexed debt			Difference between the specificationsa		
c coefficient =	1	c*	*c* coefficient =	1	c*
Costa Rica	93	63	Chile	–4	–14
Spain	88	63	Costa Rica	3	–9
Netherlands	94	73	Australia	–5	–8
Brazil	82	61	Brazil	1	–7
France	95	79	Lebanon	–7	–7
Chile	91	77	Belgium	0	–5
United States	94	82	Greece	–3	–4
South Africa	96	84	United States	2	–3
United Kingdom	96	88	United Kingdom	0	–3
Belgium	97	90	Italy	–3	–3
Indonesia	92	85	Egypt	–2	–3
Italy	86	80	France	4	–2
Australia	90	84	Malta	–1	–2
Japan	80	76	Netherlands	–1	–1
Peru	97	93	South Africa	2	–1
Israel	94	91	Indonesia	2	–1
Egypt	72	69	Austria	1	0
Austria	93	91	Israel	2	0
Germany	96	94	Japan	2	1
Greece	72	70	Spain	–1	1
Sweden	99	98	India	1	1
Malta	91	90	Cameroon	1	1
Portugal	87	86	Sweden	2	2
Argentina	88	87	Korea	1	2
Mexico	95	94	Colombia	3	3
Lebanon	58	57	Germany	5	4
India	92	91	Canada	1	4
Canada	95	95	Portugal	2	4
Turkey	96	96	Peru	3	4
Colombia	94	94	Argentina	8	7
Korea	99	99	Turkey	2	9
Cameroon	99	99	Mexico	11	15

c* = optimal indexation coefficient; iid = independent and identically distributed; VAR = vector autoregression

a. A negative value means that the reduction in the variance of the indexed debt distribution is more important under the VAR specification. Conversely, a positive value means that the reduction in the variance of the indexed debt distribution is more important under the iid shocks specification.

Source: Author's calculations.

Table A2.3 Regressions of the primary balance as a share of GDP on nominal growth rate (1996–2016)

Regressors	Argentina	Australia	Austria	Belgium	Brazil	Cameroon	Canada	Chile	Colombia	Costa Rica	Egypt	France	Germany	Greece	India	Indonesia
y	−0.039 [0.037]	0.430** [0.184]	0.252 [0.208]	0.906*** [0.180]	0.445*** [0.102]	0.415 [1.344]	0.462*** [0.127]	0.440*** [0.135]	0.004 [0.065]	0.381*** [0.047]	0.187*** [0.055]	0.687*** [0.056]	0.286* [0.147]	0.165* [0.095]	0.175 [0.132]	0.037 [0.037]
Constant	0.587 [0.633]	−2.986*** [0.957]	−0.917 [0.802]	−0.541 [0.704]	−2.932** [1.169]	0.161 [6.944]	−0.749 [0.512]	−2.792** [1.261]	0.3 [0.790]	−5.330*** [0.615]	−7.036*** [0.897]	−3.083*** [0.200]	−0.151 [0.394]	−1.592* [0.851]	−6.043*** [1.633]	0.435 [0.732]
Observations	21	21	21	21	14	17	21	21	21	21	15	21	21	21	21	21
R-squared	0.04	0.19	0.10	0.30	0.68	0.01	0.23	0.40	0.00	0.76	0.44	0.57	0.12	0.10	0.10	0.06

Regressors	Israel	Italy	Japan	Korea	Lebanon	Malta	Mexico	Netherlands	Peru	Portugal	South Africa	Spain	Sweden	Turkey	United Kingdom	United States
y	0.343*** [0.115]	0.427*** [0.112]	0.502*** [0.167]	0.293** [0.124]	−0.099 [0.205]	0.034 [0.095]	0.096*** [0.019]	0.803*** [0.084]	0.300*** [0.084]	0.249* [0.138]	0.554*** [0.106]	0.982*** [0.077]	0.398** [0.159]	0.086*** [0.024]	0.972*** [0.138]	0.997*** [0.247]
Constant	-1.337** [0.621]	0.938** [0.364]	-5.711*** [0.417]	−0.335 [0.913]	0.031 [1.485]	−0.564 [0.757]	−0.786* [0.417]	−2.757*** [0.451]	-1.107 [0.668]	-2.922*** [0.872]	-4.653*** [1.144]	-5.646*** [0.533]	−0.379 [0.636]	0.357 [0.657]	−5.976*** [0.701]	-7.100*** [1.276]
Observations	17	21	21	21	21	17	21	21	17	21	17	21	21	17	21	16
R-squared	0.21	0.34	0.23	0.34	0.02	0.01	0.34	0.83	0.37	0.13	0.58	0.92	0.21	0.42	0.39	0.36

*** p<0.01, ** p<0.05, * p<0.1

y = nominal growth rate

Note: Robust standard errors in brackets. Shaded cells indicate R^2 statistics above 0.60.

Sources: International Monetary Fund *World Economic Outlook* database, April 2017; and author's calculations.

Table A2.4a Regression of the primary balance as a share of GDP on the nominal growth rate and the output gap as a share of potential GDP (1996–2016)

Regressors	Australia	Austria	Belgium	Canada	France	Germany	Greece	Italy	Japan	Korea	Malta	Netherlands	Portugal	Spain	Sweden	United Kingdom	United States
y	0.037 [0.290]	0.035 [0.292]	0.818** [0.337]	0.238 [0.252]	0.534*** [0.149]	−0.137 [0.194]	0.310*** [0.057]	0.546*** [0.135]	0.128 [0.234]	0.363** [0.149]	0.046 [0.101]	0.793*** [0.121]	0.405** [0.171]	1.028*** [0.076]	0.04 [0.170]	0.597*** [0.174]	0.062 [0.366]
Gap	1.729*** [0.585]	0.369* [0.209]	0.2 [0.598]	0.613 [0.502]	0.268 [0.236]	0.813*** [0.254]	−0.486*** [0.089]	−0.318 [0.190]	0.699*** [0.203]	−0.14 [0.236]	−0.09 [0.149]	0.024 [0.146]	−0.359* [0.202]	−0.066 [0.106]	0.512*** [0.119]	0.854** [0.348]	1.239*** [0.288]
Constant	−0.371 [1.780]	−0.111 [1.075]	−0.216 [1.191]	0.277 [0.797]	−2.425*** [0.471]	1.115 [0.665]	−2.097*** [0.456]	0.232 [0.612]	−4.463*** [0.535]	−0.821 [0.935]	−0.744 [0.970]	−2.715*** [0.598]	−3.958*** [1.161]	−5.864*** [0.485]	1.355* [0.649]	−4.278*** [0.832]	−3.097* [1.623]
Observations	21	21	21	21	21	21	21	21	21	21	17	21	21	21	21	21	16
R-squared	0.49	0.23	0.31	0.33	0.60	0.47	0.73	0.44	0.53	0.35	0.02	0.83	0.22	0.92	0.44	0.51	0.72

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

y = nominal growth rate; gap = output gap

Note: Robust standard errors in brackets. Shaded cells indicate R^2 statistics above 0.60.

Sources: International Monetary Fund *World Economic Outlook* database, April 2017; and author's calculations.

Table A2.4b Regressions of the primary balance as a share of GDP on the output gap as a share of potential GDP (1996–2016)

Regressors	Australia	Austria	Belgium	Canada	France	Germany	Greece	Italy	Japan	Korea	Malta	Netherlands	Portugal	Spain	Sweden	United Kingdom	United States
Gap	1.786*** [0.333]	0.391** [0.170]	1.104** [0.453]	0.851** [0.302]	0.720*** [0.189]	0.698*** [0.166]	–0.372*** [0.114]	0 [0.156]	0.772*** [0.103]	0.279 [0.177]	–0.075 [0.137]	0.798*** [0.184]	0.002 [0.129]	0.650*** [0.147]	0.537*** [0.095]	1.301*** [0.343]	1.277*** [0.218]
Constant	–0.141 [0.342]	0.011 [0.235]	2.666*** [0.561]	1.337** [0.523]	–0.514** [0.242]	0.748** [0.270]	–1.078* [0.568]	2.039*** [0.374]	–4.311*** [0.412]	1.648*** [0.332]	–0.44 [0.573]	0.363 [0.423]	–2.023*** [0.542]	–1.173 [0.708]	1.534*** [0.333]	–1.747*** [0.542]	–2.846*** [0.463]
Observations	21	21	21	21	21	21	21	21	21	21	17	21	21	21	21	21	16
R-squared	0.49	0.23	0.18	0.29	0.42	0.46	0.42	0.00	0.52	0.13	0.01	0.37	0.00	0.40	0.44	0.41	0.72

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

y = nominal growth rate; gap = output gap

Note: Robust standard errors in brackets. Shaded cells indicate R^2 statistics above 0.60.

Sources: International Monetary Fund *World Economic Outlook* database, April 2017; and author's calculations.

Table A2.5 Regressions of the AMECO effective nominal interest rate on the nominal rate approximation computed using IMF WEO data

	Austria	Belgium	Bulgaria	Canada	Croatia	Cyprus	Czech Republic	Denmark	Estonia	Finland	France
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Variables	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco
i	1.214***	1.001***	0.259	0.963***	0.853***	1.347***	1.735***	0.830***	0.137	−0.511***	1.126***
	[0.067]	[0.009]	[0.156]	[0.238]	[0.193]	[0.187]	[0.443]	[0.179]	[0.136]	[0.085]	[0.015]
Constant	0.25	0.424***	4.148***	6.176***	1.218	−0.257	0.16	3.098***	4.271***	5.430***	−0.146**
	[0.237]	[0.041]	[0.480]	[1.113]	[0.835]	[0.683]	[1.211]	[0.392]	[0.608]	[0.474]	[0.056]
Observations	23	23	20	30	18	23	23	18	23	38	23
R-squared	0.94	0.998	0.133	0.368	0.55	0.711	0.422	0.573	0.046	0.501	0.996

	Germany	Greece	Hungary	Iceland	Ireland	Italy	Japan	Latvia	Lithuania	Luxembourg	Malta
	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
Variables	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco
i	1.037***	0.902***	1.202***	−0.623**	0.872***	1.113***	2.036***	0.513***	0.835**	−0.210***	0.902***
	[0.013]	[0.014]	[0.025]	[0.273]	[0.115]	[0.021]	[0.086]	[0.150]	[0.347]	[0.031]	[0.027]
Constant	0.384***	0.375***	−0.618***	9.860***	1.232***	−0.270**	0.756***	2.881***	1.467	2.111***	0.493***
	[0.049]	[0.082]	[0.187]	[1.108]	[0.422]	[0.109]	[0.136]	[0.712]	[1.555]	[0.344]	[0.143]
Observations	23	23	23	20	23	24	37	20	19	23	19
R-squared	0.997	0.995	0.991	0.225	0.733	0.992	0.942	0.394	0.255	0.682	0.985

	Montenegro	Netherlands	Norway	Poland	Portugal	Romania	Slovenia	Spain	Sweden	United Kingdom	United States
	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)
Variables	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco	iameco
i	0.457**	1.185***	−0.229	1.011***	1.179***	1.026***	2.930***	1.052***	1.124***	1.425***	1.601***
	[0.160]	[0.047]	[0.401]	[0.003]	[0.038]	[0.088]	[0.305]	[0.046]	[0.056]	[0.113]	[0.070]
Constant	2.087***	0.413**	2.284	−0.078***	−0.375**	0.888	−5.808***	0.468**	1.850***	0.016	0.433**
	[0.572]	[0.164]	[2.540]	[0.022]	[0.175]	[0.664]	[1.351]	[0.192]	[0.139]	[0.435]	[0.196]
Observations	13	23	18	23	23	18	23	23	23	26	17
R-squared	0.427	0.968	0.02	1	0.978	0.894	0.815	0.961	0.951	0.869	0.972

*** p<0.01, ** p<0.05

i = nominal rate approximation computed using IMF WEO data; iameco = AMECO effective nominal interest rate; IMF = International Monetary Fund; WEO = World Economic Outlook; AMECO = annual macroeconomic database of the European Commission

Note: Standard errors in brackets. Pink shaded cells indicate R-squares above 0.9; yellow shaded cells indicate R-squares above 0.8.

Sources: AMECO; International Monetary Fund *World Economic Outlook* database, April 2017; and author's calculations.